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THE *ARGUMENTA* OF DIONYSIUS EXIGUUS AND THEIR EARLY RECENSIONS

Abstract

Dionysius Exiguus composed the earliest known computistical formulary written in Latin in 525. However, this formulary has not survived in its original form. The editors of Dionysius' computistical writings, Wilhelm Jan and Bruno Krusch, published a corpus of 16 *argumenta* from a single manuscript, namely MS Oxford, Bodleian Library, Digby 63 under Dionysius' name, since this was the only manuscript known to them that preserved the original 525 dating. Some of these 16 *argumenta*, however, contain dating clauses as late as 675, which immediately cast doubt on their ascription to Dionysius. In fact, the 16 *argumenta* edited by Jan and Krusch should more precisely be defined as a computistical formulary of 675, to be termed the *Computus Digbaeanus* of 675, which includes the original Dionysiac *argumenta.* This article, then, reconstructs the original Dionysiac corpus on the basis of new manuscript evidence. Moreover, the different stages of interpolations and additions that eventually led to the composition of the *Computus Digbaeanus* are analysed, and with this the development of computistical formularies written in Latin in the 150 years from 525 to 675.

Keywords

Dionysius Exiguus, Cassiodorus, Maximus Confessor, Bede, Hrabanus Maurus, history of computus, computistical formularies, computistical *argumenta, Computus paschalis* of 562*, Computus Digbaeanus* of 675*, Computus Cottonianus* of 688/9*, Computus Rhenanus* of 775*, Fragmentum Nanciacense*, Sirmond manuscript, Irish computistics, Anglo-Saxon computistics, Frankish computistics.

Introduction

In the early centuries of Christianity, the West and the East followed different Easter cycles, which led to conflicting dates for Easter Sunday and continuous disputes between Rome and Alexandria.[1] An attempt to reconcile the two systems had failed in the mid-fifth century, when Victorius of Aquitaine was order by the papal curia to establish an Easter table that would solve the differences.[2] Victorius' system was finally approved in Rome, and it became very popular in the West, especially the Frankish kingdoms,[3] but his decision to list two possible dates for Easter Sunday in certain years did not solve the problem, while even his unambiguous dates did not always correspond to the Alexandrian ones.[4] Then, in 526, Easter Sunday was supposed to fall on *luna* 22 according to Victorius,[5] which was regarded as an unacceptable lunar age in the Alexandrian reckoning. It was presumably for that reason that the papal curia decided in the previous year, 525, to contact Dionysius Exiguus about this question.[6] By this time, Dionysius had made a name for himself as a canonist and translator from Greek into Latin,[7] and it probably was precisely his reputation as

[1] For the early history of the Easter controversy (technical and historical) see Krusch (1880); Schwartz (1905); Mac Carthy (1901), xxxii–clxxvii; Ideler (1825–26), ii 191–275; Schmid (1907); Ginzel (1914), 210–20, 232–45; Rühl (1897), 107–126; Gentz in PW 18, 1647–53 (*s.v.* Ostern); Chaîne (1925), 19–61; Jones (1943), 6–77; Strobel (1977); Wallis (1999), xxxiv–l; Declercq (2000), 49–82; van de Vyver (1957); Grumel (1960); Lejbowicz (2006), 10–63.

[2] Victorius' Easter table and prefixed prologue are edited by Krusch (1938), 4–52. For the Victorian reckoning see Ideler (1825–6), ii 275–84; Schwartz (1905), 72–80; Ginzel (1914), 245–7; Rühl (1897), 126–8; Jones (1943), 61–8; Wallis (1999), l–lii; Declercq (2000), 82–95; Declercq (2002), 181–7.

[3] The most explicit source for the adoption of the Victorian system can be found for the Frankish kingdoms in form of the Acts of the Council of Orleans of 541 (*Concilium Aurelianense a.* 541 §1: CCSL 148A, 132; MGH Conc. 1, 87): *Placuit itaque Deo propitio, ut sanctum pascha secundum laterculum Victori ab omnibus sacerdotibus uno tempore celebretur.* For the adoption and use of the Victorian system in the Latin West see Krusch (1884); Poole (1918a); Schmid (1907), 38–107; Jones (1934), 412–20; Jones (1943), 65–6.

[4] For the differences between the two systems see especially Schwartz (1905), 73–80; Ginzel (1914), 246; Jones (1934), 411–2.

[5] Krusch (1938), 51.

[6] Cf. Krusch (1884), 107–8; Krusch (1938), 59; Ginzel (1914), 247; Jones (1934), 414; Jones (1943), 68; Borst (1998), 177; Wallis (1999), liii; Declercq (2000), 107–9; Declercq (2002), 203–5.

[7] For Dionysius' life and work see Jülicher in PW 5, 998–9 (*s.v.* Dionysius Exiguus); Mordek in LM 3, 1088–92 (*s.v.* Dionysius Exiguus). For Dionysius' origin being Alexandrian/ Egyptian rather than Scythian see Zeller (1991), 167–8.

translator that persuaded the papal curia to ask for his help: In the end, the task to be accomplished was to translate the Alexandrian Easter table and instructions from Greek into Latin.[8] Dionysius composed his computistical works in two stages, and it is only the first that will concern us here:[9] 1) He translated, converted, transformed, and continued the 95-year Easter table of Cyril of Alexandria for the subsequent 95-year period, from 532 to 626, providing a manual for this table in a prologue addressed to an unidentified bishop Petronius, and adding a Latin translation of the letter of Proterius and a series of *argumenta.* 2) After Dionysius had accomplished this task, the letter of Paschasinus was found in the papal library, which made the papal curia wonder about the workings of the 19-year lunisolar-cycle. Again, they consulted Dionysius about this problem, and his reply in form of a letter of 526 marks the second stage.

Now, the focus of the present article is on the above-mentioned *argumenta* that Dionysius attached to his Easter table. Dionysius introduced them in his prologue in the following way:[10]

> *Nec non et argumenta Aegyptiorum sagacitate quaesita subdidimus, quibus, si forsitan ignorentur, paschales tituli possint facile repperiri; id est, quotus annus sit ab incarnatione domini et quae sit indictio, quotus etiam lunaris circulus, decennovenalis existat ceteri aeque simili supputationis compendio requirentur.*
>
> 'We have also supplied *argumenta*, which we obtained through the acuteness of the Egyptians, with whose aid the data of the columns of the Easter table can easily be found, should they perhaps not be known; namely, the year which happens to be from the incarnation of the Lord, and which indiction this year happens to have, and also which year in the *cyclus lunaris* and *decemnovennalis* it happens to be; and the data of the other columns should likewise be sought by similar computational methods.'

It is very unfortunate that Dionysius mentions only four *argumenta* explicitly in this paragraph and does not supply the reader with a comprehensive list. Such a list of all *argumenta* compiled by Dionysius in 525 would have saved modern scholars much trouble,

[8] Cf. Neugebauer (1979), 101–5; Neugebauer (1982).

[9] Dionysius' computistical writings were edited by Jan (1718), republished in PL 67, 453–520, and by Krusch (1938), 59–86.

[10] Krusch (1938), 67. The translation is mine. A different translation of this passage can be found in Teres (1984), 181.

since the manuscript transmission of these *argumenta* is not straightforward. Only two modern editions of them exist, one published by Wilhelm Jan in 1718, the other by Bruno Krusch in 1938;[11] both scholars edited these *argumenta* on the basis of a single manuscript, namely MS Oxford, Bodleian library, Digby 63, because this was the only manuscript known to them that preserved the 525 dating. Yet, this corpus of 16 *argumenta* published by Jan and Krusch contains only nine with dating clauses for 525, whereas the remaining seven have either no or later dating clauses, the latest for 675. Still, Jan, and Krusch following him, decided to publish the entire corpus among the computistical works of Dionysius Exiguus. This corpus of 16 *argumenta*, preserved uniquely in MS Oxford, Bodleian Library, Digby 63, 72v–79r, will be referred to as the *Computus Digbaeanus* of 675 in the following.[12]

The *Computus Digbaeanus* is one of the earliest known corpora of computistical formulae and / or short texts (which I will term a computistical formulary in the following) written in Latin. The earliest is, in fact, the original corpus of the Dionysiac *argumenta.* Unfortunately, the Dionysiac corpus has not survived in its original form. The only other known formulary from the sixth century is the so-called *Computus paschalis* of 562, controversially attributed to Cassidorus.[13] This formulary seems to be nothing but original Dionysiac *argumenta* with the examples being accommodated to match the chronological data of 562. Consequently, the *Computus paschalis* of 562 transmits valuable information about the extent of the original Dionysiac corpus. From the entire seventh century, the *Computus Digbaeanus* of 675 and the so-called *Computus Cottonianus* of 688/9 are the only known formularies.[14] Interestingly

[11] See n 9.

[12] For this text see pp. 3–8 below.

[13] For this text see p. 18 below.

[14] For the *Computus Cottonianus* see pp. 22–3 below. Some of the *argumenta* of the *Computus Cottonianus* with seventh-century dating clauses found their way into the *Computus Rhenanus* of 775 and from there into the tenth century *Canones lunarium decemnovennalium circulorum* (cf. pp. 23–4, especially n 111 below). Other than that, I am presently aware of only one further manuscript that contains *argumenta* with seventh-century dating clauses, namely MS München, Bayerische Staatsbibliothek, Clm 14725: a section on folio 14r-v consists of the Dionysiac *Argumenta I*, *II*, *V*, and *VIII* with explicit dating clauses for 695. These should be compared with the *Computus Cottonianus* and the *Computus Rhenanus.* The three *argumenta* of the *Fragmentum Nanciacense* with dating clauses for 625 were later incorporated into the *Computus Digbaeanus* and are discussed in that context below (cf. pp. 19–21, 32, 35).

enough, the *Computus Digbaeanus* includes all the *argumenta* of the original Dionysiac corpus, as far as can be established, and consequently also of the *Computus paschalis*; in fact, it preserves all known *argumenta* with dating clauses for 675 or earlier. Therefore, a study of the *Computus Digbaeanus* actually results in a study of the development of computistical formularies written in Latin over the first 150 years, i.e. from 525 to 675.

Additionally, the *Computus Digbaeanus* marks a watershed in the development of computistical formularies: The number of only 16 *argumenta* preserved in the *Computus Digbaeanus* is an impressive witness to the slow process of development in this field over the first 150 years. However, this process accelerated rapidly in the last quarter of the seventh century and then especially in the eighth and ninth centuries, a development in which the *Computus Digbaeanus* played a major part: It had opened up the Dionysiac corpus which was previously strictly defined. This allowed further formulae and texts to be added to that corpus, which were invented in considerable numbers once the Dionysiac reckoning was introduced into Ireland and Britain, and later into the Frankish kingdoms. In this process the original Dionysiac *argumenta* lost their attribution and thus their authoritative connotation, which led to them being revised either by the addition of further explanations, or by generalisation; due to this revision they lost their original phrasing and shape, while the corpus itself disintegrated with selected *argumenta* being incorporated in larger computistical formularies or compendia.

It is the purpose of this article, then, to analyse the different stages in the development of computistical formularies before this significant change occurred, i.e. in the 150 years between 525 and 675, by means of examining the different layers of the *Computus Digbaeanus.* For this, it is at first necessary to introduce the 16 *argumenta* of the *Computus Digbaeanus* at some length as the basis of discussion, and at the same time the final stage of the development in question. The starting point of this development, i.e. the original corpus of Dionysiac *argumenta*, is, however, highly disputed. The different views on this first computistical formulary in Latin will be reviewed, before an attempt is made to reconstruct that corpus. From that basis it will be possible to trace the intermediary stages in the development from the original corpus of Dionysiac *argumenta* of 525 to the *Computus Digbaeanus* of 675. Concerning terminology, of the 16 *argumenta* in question, the ones that are likely to have been compiled by Dionysius himself are termed the

Dionysiac *argumenta*,[15] the ones that were later added are called the pseudo-Dionysiac *argumenta.*

Description of the Computus Digbaeanus of 675

The *Computus Digbaeanus* of 675 comprises folios 72v–79r of the ninth-century MS Oxford, Bodleian Library, Digby 63; its provenance is highly disputed.[16] The beginning of this computus is well defined, starting in the lower quarter of folio 72v, with the rest of the page being left blank. Moreover, a title is given: *Incipiunt argumenta de titulis pascalis egiptiorum investigata solercia ut praesentes indicent.* The end of this computus, however, can less easily be established. Jan and Krusch in their editions believed that the *Disputatio Morini*, starting on folio 79r, marked the end, arguing that this computus comprises 16 *argumenta.* But it should be noted that at least the inclusion of *Argumentum XVI* is debatable, since it is the only one of these 16 that has a separate heading (*De racione bissexti*), and may thus have been regarded as an independent treatise. The 16 *argumenta* in question have often been described, but the mathematical formulae have not yet been satisfactorily explained in print.[17] A proper understanding of these *argumenta* is, however, vital for their assessment:

Argumentum I is divided into three paragraphs, of which only the first is printed in the main body of the editions;[18] §1 gives a formula for

[15] This article will not assess the question whether Dionysius is rightly credited with the composition of these *argumenta*, or whether he should only be credited with their translation, which certainly is a matter of debate. From the citation given above it seems, however, that Dionysius acted merely as translator. For this question see especially Neugebauer (1982); Pallarès (1994), 16; Declercq (2002), 201–2.

[16] Cf. Dumville (1981), 168; Dumville (1983); Ó Cróinín (1982b), 407; Ó Cróinín (1983d), 257–9. For catalogue descriptions of this MS see Hunt and Watson (1999), 64–6; Lindsay (1915), 470; Krusch (1926), 51–4; Jones (1939), 127; Borst (2001), 161–2; Borst (2006), 263–4. See also Englisch (2002), 127.

[17] For discussions and mathematical notations of these *argumenta* see Springsfeld (2002), 172–182 (which is sometimes faulty); Neugebauer (1982) (where *Argumenta I–VI, VIII–XI* are explained as part of the *Computus paschalis* of 562); Pedersen (1983), 53 (who only treats *Argumenta II–V*). The Latin text of these *argumenta* with English and German translation respectively, as well as valuable mathematical commentary, is provided by Michael Deckers and Nikolaus A. Bär on the world wide web: http://hbar.phys.msu.su/gorm/chrono/paschata.htm and http://www.nabkal.de/dionys.html.

[18] §1: Krusch (1938), 75 ll. 1–4; § 2: Krusch (1938), 75 variant c ll. 1–3 (ending with *conputa*); §3: Krusch (1938), 75 variant c ll. 3–9 (starting with *item ad aliam*).

calculating the AD date from the indiction; the example given is AD 525, the general formula:

$15 \times 34 + 12 + \text{indiction of the present year} = \text{AD}$

It is composed of the following parameters: the fixed parameter 12 indicates the number of years from Christ's incarnation to the end of an indiction cycle (AD 12=Indiction 15); from AD 12 to AD 522 another 34 indiction cycles have passed, hence 15×34; thus, the indiction of any given year between 523 and 537 added to the sum $12+15 \times 34$ equals the number of years from the incarnation to the year in question.

It is obvious that the number of full indiction cycles between AD 12 and the year in question increases by one every 15 years, and the formula has to be adjusted accordingly, namely by increasing the multiplicand in the product 15×34 by one every 15 years (e.g. 34 is replaced by 35 in 537). This provision is taken in §§2 and 3, which are omitted in Jan's edition, and relegated to the apparatus in Krusch's.[19] Note that in a year with indiction 15 either this number can be added as the indiction of the present year or the multiplicand with 15 can be increased by one, which leaves indiction 0 for the present year; the latter option is chosen in §3.

Argumentum II gives a formula for calculating the indiction from the AD date; the example given is AD 525, the general formula:

$(\text{AD}+3) \bmod 15 = \text{indiction}$ (y mod z denotes the remainder after division of y by z)

The fixed parameter 3 is chosen because it is obviously necessary for this calculation that the count of years starts with the beginning of an indiction cycle; AD 1, however, has indiction 4, so another 3 are added to the AD date (BC 3=indiction 1); the modulo15 calculation, then, eliminates all full indiction cycles, with the remainder being the indiction of the AD date in question.

Argumentum III is divided into two paragraphs.[20] §1 gives a formula for calculating the epact, i.e. the lunar age of 22 March, from the AD date; the example given is AD 525, the general formula:

$(\text{AD} \bmod 19 \times 11) \bmod 30 = \text{epact}$

The logic behind this calculation is this: BC 1 is the first year of a *cyclus decemnovennalis* (i.e. the 19-year luni-solar cycle underlying the Dionysiac reckoning); consequently, the remainder of the AD date

[19] For these two paragraphs see the previous note and the discussion on pp. 26–7 below.

[20] §1: Krusch (1938), 75 ll. 9–12; §2: Krusch (1938), 75 l. 13 – 76 l. 3.

divided by 19 (ADmod19) equals the number of years from the first year of the *cyclus decemnovennalis* in which this AD date occurs; since the first year of the *cyclus decemnovennalis* has epact 0, and since the epact increases by 11 per year in this 19-year cycle, ADmod19 is multiplied by 11; the modulo30 calculation then eliminates all intercalated lunar months of 30 days, with the remainder being the epact of the year in question.

§2 gives a formula for calculating the epacts from the year in the *cyclus decemnovennalis*;[21] the examples given are the 10th and 11th year of the *cyclus decemnovennalis*, neither of which agrees with AD 525, since cyclic number 10 corresponds to $522+n\times19$; the formula is:

((year in the *cyclus decemnovennalis*$-1)\times11)$mod30=epact

The logic is the same as for the formula in §1: the first year of the *cyclus decemnovennalis* has epact 0, hence the subtraction of 1; the epact increases by 11 per year in this 19-year cycle, hence the multiplicand 11; the modulo30 calculation then eliminates the intercalated lunar months of 30 days.

Argumentum IV is also divided into two paragraphs.[22] §1 gives a formula for calculating the *concurrentes*, i.e. the weekday of 24 March, from the AD date; the example given is AD 525, the formula:

(AD+[AD/4]+4)mod7=*concurrentes* ([y] denotes the greatest integer less than or equal to y)

The logic behind this formula is this: AD 1 has *concurrentes* 5; since the *concurrentes* increase by one every year, the number of years from AD 1 (AD−1) are added to the *concurrentes* of that year (5), resulting in AD−1+5=AD+4; hence the parameter 4; every fourth year, however, the *concurrentes* increases by 2 instead of 1 due to the bissextile day, hence the parameter [AD/4] (i.e. an extra day is added for every fourth year); since the *concurrentes* denote a weekday, and therefore never exceed 7, the modulo7 calculation is then applied, with the remainder being the *concurrentes* in question.

§2 intended to simplify the calculation by counting the years not from the incarnation, but from the year following the consulship of Tiberius Iunior Augustus.[23] It is argued that the fixed parameter had then to be changed from 4 to 1, thereby implying that the first year of the new count has *concurrentes* 2. Moreover, since no parameter is

[21] For this paragraph see pp. 27–8 below.

[22] §1: Krusch (1938), 76 ll. 4–10; §2: Krusch (1938), 76 ll. 11–14.

[23] For this paragraph see p. 28 below.

added to the number of years to be divided by 4, the first year of the new count has to be the first year in a four-year bissextile cycle, i.e. a year immediately following a bissextile year. These chronological data match AD 581.

Argumentum V gives a formula for calculating the year in the *cyclus decemnovennalis* from the AD date; the example given is AD 525, the formula:

$(\mathrm{AD}+1)\mathrm{mod}19=$ year in the *cyclus decemnovennalis*

The fixed parameter 1 is chosen, since it was obviously necessary for this calculation that the count of years starts with the first year of a *cyclus decemnovennalis*; AD 1, however, is the second year in such a cycle, and thus 1 is added to the AD date (BC 1=first year in the *cyclus decemnovennalis*); the modulo19 calculation, then, eliminates all full 19-year cycles, with the remainder being the year in the *cyclus decemnovennalis* in question.

Argumentum VI gives a formula for calculating the year in the *cyclus lunaris* from the AD date; the example given is AD 525, the formula:

$(\mathrm{AD}-2)\mathrm{mod}19=$ year in the *cyclus lunaris*

Again, the count of years obviously has to start with the first year of a *cyclus lunaris*; hence the fixed parameter −2, since a *cyclus lunaris* does not start in AD 1, but two years later, in AD 3; the modulo19 calculation the, eliminates all full 19-year cycles, with the remainder being the year in the *cyclus lunaris* in question.

Argumentum VII lists the seven years of the *cyclus decemnovennalis*, in which the Easter full moon falls in March; in the remaining twelve, the Easter full moon falls in April.

Argumentum VIII gives a formula for establishing whether or not a year is bissextile, calculated from its AD date; the example given is AD 525, the formula:

$\mathrm{AD}\mathrm{mod}4=y$; if $y=0$, then AD is bissextile

The logic behind this formula obviously is that every AD date divisible by 4 is bissextile.

Argumentum IX can be divided into two paragraphs.[24] §1 gives a formula for calculating the lunar age of Easter Sunday from the epact and the Julian calendar date of Easter Sunday; the two examples given are: 1) a year with indiction 3, epact 12, Easter Sunday on 30 March,

[24] §1: Krusch (1938), 77 ll. 5–15; §2: Krusch (1938), 77 ll. 16–19.

luna 20, and 2) a year with indiction 4, epact 23, Easter Sunday on 19 April, *luna* 21; these examples agree with AD 525 and 526 respectively; the general formula is:

- **a)** Easter Sunday falling on y March: (8+epact+y)mod30=lunar age of Easter Sunday
- **b)** Easter Sunday falling on z April: (9+epact+z)mod30=lunar age of Easter Sunday

These two formulae can be explained in the following way:

- **a)** If Easter Sunday falls on y March, then the Julian calendar day difference between y March and 22 March, the date of the epact, equals the lunar day difference between the lunar age on Easter Sunday and the epact; in mathematical terms: y−22=lunar age of Easter Sunday−epact; this is equivalent to y−22+epact=lunar age of Easter Sunday; since the lunar age of Easter Sunday ranges between 1 and 30, this is equivalent to (−22+epact+y)mod30=lunar age of Easter Sunday; since (−22)mod30≡8mod30, the equation is equivalent to (8+ epact+y)mod30=lunar age of Easter Sunday.
- **b)** If Easter Sunday falls on z April, then the Julian calendar day difference between z April and 22 March, the date of the epact, is congruent modulo30 to the lunar day difference between the lunar age on Easter Sunday and the epact (the modulo30 congruence is necessary here, since the epact and the lunar age of Easter Sunday may occur in two successive lunations, with the lunation ending between them consisting of 30 lunar days); in mathematical terms: (z+9)mod30≡(lunar age of Easter Sunday−epact)mod30; this is equivalent to (z+9+epact) mod30≡(lunar age of Easter Sunday)mod30; since the lunar age of Easter Sunday naturally ranges between 1 and 30, this leads to the equation (z+9+epact)mod30=lunar age of Easter Sunday.

§2 then seems to supply a confused explanation of the parameters 8 and 9 fixed for March and April respectively.[25]

Argumentum X can also be divided into two paragraphs.[26] §1 gives a formula for calculating the weekday of any given Julian calendar date

[25] Cf. pp. 29–30 below.

[26] §1: Krusch (1938), 77 ll. 20–25; §2: Krusch (1938), 77 ll. 25–27.

from the *concurrentes* of that year; the example given is a year with indiction 3, *concurrentes* 2, and Easter Sunday on 30 April; these data agree with AD 525; the general formula is:

Let y be the inclusive number of days from 1 January to the Julian calendar date in question; then $(y+1+\textit{concurrentes})\bmod 7=$ weekday of that Julian calendar date.

This equation can be explained in the following way: Let z be the weekday on 1 January in the year in question; $y-1$ denotes the exclusive number of days from 1 January to the Julian calendar date in question; then $(z+y-1)\bmod 7=$ weekday of the Julian calendar date in question; the exclusive number of days from 1 January to the place of the *concurrentes*, 24 March, is 82, and since $82\bmod 7=5$, $(z+5)\bmod 7=\textit{concurrentes}$; this is equivalent to $z=(\textit{concurrentes}-5)\bmod 7$, which is equivalent $z=(\textit{concurrentes}+2)\bmod 7$ (since $-5\bmod 7\equiv 2\bmod 7$); now, substituting z in the equation above with this result leads to $(\textit{concurrentes}+y+1)\bmod 7=$ weekday of the Julian calendar date in question.

§2 then explicitly states that this formula can be applied for any given Julian calendar date of a year.

Argumentum XI gives a formula for calculating the epact from the AD date; the example given is AD 675, the formula:

$(\mathrm{AD}\bmod 19\times 11)\bmod 30=\text{epact}$

The formula is exactly the same as in *Argumentum III* above.

Argumentum XII gives a formula for calculating the weekday of 1 January from the AD date; the example given is 675, the formula:

$(\mathrm{AD}-1+[(\mathrm{AD}-1)/4])\bmod 7=\text{weekday on 1 January}$

The method of this *argumentum* is basically the same as in *Argumentum IV*, with some important variations: The weekday of 1 January increases by one every year; hence the number of years from AD 1 $(\mathrm{AD}-1)$ are added to the weekday on 1 January of AD 1, which is *feria* 7; in every bissextile year the weekday increases by 2 instead of 1; the bissextile day occurs in every AD divisible by 4; but since the bissextile day occurs after 1 January, the two day increase on that date happens only in the following year; hence the parameter $[(\mathrm{AD}-1)/4]$, which provides for an extra day being added for the years following a bissextile year; since weekdays never exceed 7, the modulo7 calculation is applied, which leads to the equation: $(\mathrm{AD}-1+7+[(\mathrm{AD}-1)/4])\bmod 7=$ weekday on 1 January; since $7\bmod 7\equiv 0\bmod 7$, this is equivalent to $(\mathrm{AD}-1+[(\mathrm{AD}-1)/4])\bmod 7=$ weekday on 1 January.

Argumentum XIII gives a formula for calculating the lunar age of 1 January from the year in the *cyclus lunaris*; the two examples given are the 15^{th} and 17^{th} year of the *cyclus lunaris*, having *luna* 16 and 9 on 1 January respectively; these data do not agree with AD 675, but with AD 625 and 627, as will be seen further below; the general formula is:

> (1+year in the *cyclus lunaris*×5+year in the *cyclus lunaris*×6) mod30=*luna* on 1 January

In years 17 to 19 of the *cyclus lunaris* the fixed parameter has to be increased by 1 from 1 to 2.

The logic is this: The 19^{th} year of the *cyclus lunaris* has *luna* 1 on 1 January, hence the parameter 1; the lunar age increases by 11 per year, here divided into the multiplicands 5 and 6; the modulo30 calculation is then applied to eliminate all intercalated lunar months of 30 days; the *saltus lunae* occurs at the end of the 16^{th} year of the *cyclus lunaris*, which results in an increase of 12 instead of 11 lunar days between the 16^{th} and the 17^{th} year; for that reason the fixed parameter is increased by one for the three years following the *saltus lunae.*

Moreover, the period of time between the beginning of the first hour of a day and moonrise is given in points for both dates (i.e. 1 January of the 15^{th} and of the 17^{th} year of the *cyclus lunaris*); this is discussed in detail in Appendix II.

Argumentum XIV gives a formula for calculating the Julian calendar date and weekday of the Easter full moon, as well as the Julian calendar date and lunar age of Easter Sunday, from the epact and *concurrentes* of a given year; the three examples outlined are the first three years of the *cyclus decemnovennalis* (having epacts 0, 11, and 22); the *concurrentes* assigned to these years are 4, 5, and 6 respectively, with the lunar and solar data combined agreeing with AD 532 to 534. The general formula is:

a) Easter full moon in March: 36−epact=y March for Easter full moon

> (y+*concurrentes*+4)mod7=weekday of Easter full moon

b) Easter full moon in April: (35−epact)mod30=z April for Easter full moon

> (z+*concurrentes*+7)mod7=weekday of Easter full moon

With this information it was simple to calculate the Julian calendar date and lunar age of Easter Sunday, which occurred on the Sunday following the Easter full moon.

The two formulae can be explained in the following way:

- **a)** If the Easter full moon occurs on y March, then the Julian calendar day difference between y March and 22 March, the date of the epact, equals the lunar day difference between the Easter full moon (*luna* 14) and the epact; in mathematical terms: $y-22=14-\text{epact}$; this is equivalent to $y=36-\text{epact}$, the first part of the formula; then, the Julian calendar difference between the Easter full moon (y March) and the place of the *concurrentes* (24 March) is congruent modulo7 to the weekday difference between the weekday of the Easter full moon and the *concurrentes* (the modulo7 congruence is necessary here, since the weekday of the Easter full moon and the *concurrentes* may occur in different weeks); in mathematical terms: $(y-24)\text{mod}7\equiv(\text{weekday of Easter full moon}-\textit{concurrentes})\text{mod}7$; since $-24\text{mod}7\equiv4\text{mod}7$ this is equivalent to $(y+\textit{concurrentes}+4)\text{mod}7\equiv(\text{weekday of Easter full moon})\text{mod}7$; since the weekday of the Easter full moon naturally ranges between 1 and 7, this leads to the equation $(y+\textit{concurrentes}+4)\text{mod}7=\text{weekday of Easter full moon}$, the second part of the formula.
- **b)** If the Easter full moon occurs on z April, then the Julian calendar day difference between z April and 22 March, the day of the epact, is congruent modulo30 to the lunar day difference between the Easter full moon (*luna* 14) and the epact (the modulo30 congruence is necessary here, since the epact and the Easter full moon may occur in two successive lunations, with the lunation ending between them consisting of 30 lunar days); in mathematical terms: $(z+9)\text{mod}30\equiv(14-\text{epact})\text{mod}30$; this is equivalent to $z\text{mod}30\equiv(5-\text{epact})\text{mod}30$; since z naturally does not exceed 30 (it ranges between 1 and 18) and $5\text{mod}30\equiv35\text{mod}30$, this leads to the equation $z=(35-\text{epact})\text{mod}30$, the first part of the formula; then, the Julian calendar day difference between the Easter full moon (z April) and the place of the *concurrentes* (24 March) is congruent modulo7 to the weekday difference between the Easter full moon and the *concurrentes* (again, the modulo7 congruence is necessary here, since the weekday of the Easter full moon and the *concurrentes* do certainly occur in different weeks); in mathematical terms: $(z+7)\text{mod}7\equiv(\text{weekday of Easter full moon}-\textit{concurrentes})\text{mod}7$; this is equivalent to $(y+\textit{concurrentes}+7)\text{mod}7\equiv(\text{weekday of Easter full moon})\text{mod}7$; since the weekday of the Easter

> full moon naturally ranges between 1 and 7, this leads to the equation (x+*concurrentes*+7)mod7=weekday of Easter full moon, the second part of the formula.

Argumentum XV describes the correspondence of the Roman equinoxes and solstices with the conception and birth of Christ and of John the Baptist, as well as with the passion of Christ. Moreover, it gives a chronology of Christ's life.

Argumentum XVI explains the bissextile increase of one point (defined as a quarter of an hour) per month, amounting to 12 hours in four years (i.e. the 12 hours of daytime). Moreover, it gives a curious mathematical explanation for the alledged 3 hours bissextile increase per year by dividing the 8760 hours of a year by 7, leaving a remainder of just these 3 hours.[27]

Previous Views on the Original Corpus of the Dionysiac Argumenta

Which of these *argumenta*, then, can be classified as Dionysiac, i.e. as belonging to the original corpus of *argumenta* compiled by Dionysius Exiguus himself? Scholarly opinion on this important question varies greatly:

The discussion about the Dionysiac *argumenta* really started with the *editio princeps* of the *Computus Digbaeanus* by Wilhelm Jan in 1718, which is a model of scholarship for its time, and still the best edition of Dionysiac computistica available. He justified the ascription of these *argumenta* to Dionysius in detail,[28] but at the same time emphasized that some of the 16 *argumenta* published by him on the basis of the *Computus Digbaeanus* of 675 (which he checked with the *Computus Cottonianus* of 688/9) appear to be later additions, while others appear to contain later interpolations.[29] About this pseudo-Dionysiac section he was then more specific in the notes to each *argumentum.* Jan generally applied four criteria to distinguish Dionysiac from pseudo-Dionysiac *argumenta*:[30] 1) Agreement with the columns of the Dionysiac

[27] For this calculation see especially Springsfeld (2002), 204; Springsfeld (2003), 226. Cf. also pp. 33–4 below.

[28] Jan (1718), 79–80.

[29] Jan (1718), 48–9.

[30] Jan (1718), 54, 81–8.

Easter table; 2) the reception of the *argumenta* by early medieval computists, primarily Bede; 3) the transmission of the *argumenta*; 4) internal textual evidence, including dating clauses given in the examples. The strongest of these criteria obviously is the fourth: nine of the first ten *argumenta* include examples that refer either explicitly to the year 525 (which is also the year mentioned in Dionysius' prologue), or implicitly, with the chronological data corresponding to that year. The only *argumentum* among the first ten that does not include a dating clause is *Argumentum VII*. Interestingly enough, Jan did not comment on this particular *argumentum*, and it must be presumed that he regarded it as Dionysiac. Consequently, the first ten *argumenta* are Dionysiac according to Jan. Three of these, however, contained later interpolations:[31] 1) Jan did not publish §§2 and 3 of *Argumentum I*, presumably because he regarded them as later additions; unfortunately, he did not comment on this omission, which appears strange considering the fact that these paragraphs also appear in the *Computus Cottonianus* (albeit in accommodated and altered form), which Jan consulted for comparison; 2) since §2 of *Argumentum III* does not occur in the *Computus Cottonianus*, and because it was introduced by the phrase *Item aliud computum*, Jan argued that it was not part of the original *argumentum*; 3) the same applies to §2 of *Argumentum IV*, which is introduced by the phrase *Item nuper inuentum, melius iudicaui, si breuiter patefiat*, and which again does not appear in the *Computus Cottonianus*; moreover, Jan thought that this paragraph was not transmitted in full in his *Leithandschrift*; the mentioning of the consulate of Tiberius Iunior Augustus combined with the chronological data given led Jan to date this interpolation to *c.*581.[32] In addition to these three later additions, Jan was suspicious about §2 of *Argumentum IX*, stating that it appears to be out of place here, but he left that problem for others to solve.[33]

Concerning the remaining six *argumenta*, Jan restricted himself to a general remark in a note on *Argumentum XI* rather than to comment on each of these individually, apparently because he found them difficult to understand:[34] After outlining that this and the following *argumentum* transmit dating clauses for 675 in the *Computus Digbaeanus*, for

[31] Jan (1718), 81, 83–4.

[32] It seems quite probable that Jan did not establish this date himself, but simply adopted the date written in a late hand in the margin to this paragraph in the manuscript (f 74r).

[33] Jan (1718), 87.

[34] Jan (1718), 88.

688 or 689 in the *Computus Cottonianus* (i.e. one and a half centuries later than the dating clauses found in the previous *argumenta*), he concluded very cautiously that these *argumenta* were either added or at least emended by later computists.

The most detailed discussion about the authorship of these *argumenta* was written just 26 years later by one of the most neglected scholars of late antique and early medieval computistics, Johannes van der Hagen.[35] Having no further manuscript evidence at hand, van der Hagen based his study on Jan's edition, which meant that he was not aware of §§2 and 3 of *Argumentum I.* His approach is more textual and mathematical, with the main new textual criterion introduced by him being the question of the general format of an *argumentum.* It seems that in his opinion an *argumentum* had to contain a mathematical formula. Consequently, he was very suspicious of every *argumentum* among the 16 in question that does not do so. Presumably on this ground, he regarded *Argumenta XV* and *XVI* as pseudo-Dionysiac. Concerning the only other *argumentum* of that sort, *Argumentum VII*, he did not commit himself to an explicit statement of authorship. But it appears that he regarded it as pseudo-Dionysiac, because it had no formula, and because it appears out of place among *argumenta* that otherwise show very similar patterns. On this basis of structure he divided the nine *argumenta* remaining of the ten classified by Jan as Dionysiac into two groups: The first group consists of *Argumenta I–VI* and *VIII*, for the reason that *Argumentum I* provides an algorithm for the calculation of the AD date of a year, which is then used as the sole precondition for the formulae of all of remaingn six *argumenta* of this group; in van der Hagen's opinion this group is to be classified with certainty as Dionysiac, since the examples in all of these explicitly refer to 525, i.e. the year in which Dionysius composed his prologue; concerning §2 of *Argumenta III* and *IV*, he agreed with Jan that they are later interpolations, adding that §2 of *Argumentum III* does not fit into the structure of the *argumenta* of this group, since the precondition in the formula of this paragraph is the year of the *cyclus decemnovennalis*, not the AD date; moreover, he dates §2 of *Argumentum IV* to the year 703. The second group, then, consists of *Argumenta IX* and *X.* Since these also have dating clauses for 525 (though only implicit ones), van der Hagen argues that *videntur etiam esse Dionysii.* His hesitance in attributing them to Dionysius stems from the fact that these two

[35] Van der Hagen (1734), 205–8.

argumenta do not share the structure of the previous ones, i.e. that their precondition is not the AD date.

In contrast to Jan, van der Hagen discussed in detail the six *argumenta* that Jan had classified as pseudo-Dionysiac. First he treated *Argumenta XI* and *XII* together, accepting Jan's classification as pseudo-Dionysiac on the basis of the examples explicitly referring to the year 675, while adding that *Argumentum XI* deals with a question that had already been discussed in *Argumentum III*, namely the calculation of the epact of a given year. He then connected *Argumentum XIII* with Maximus Confessors' *Computus ecclesiasticus* rather than Dionyisus' works, a very interesting observation that has escaped the notice of later scholars.[36] Turning to *Argumentum XIV*, he argued that it might have been composed by Dionyisus himself, because the chronological data given agree with the years 532–534; but since these are the first three years of the Dionysiac Easter table, van der Hagen thought that these examples may as likely have been composed by a later computist, who found these three years suitable to illustrate his calculations. Finally, he regarded *Argumenta XV* and *XVI* as pseudo-Dionysiac, because they simply did not constitute *argumenta* in his sense of the word.

In 1901, Bartholomew Mac Carthy supported Jan's and van der Hagen's view that *Argumenta XI–XVI* must be regarded as pseudo-Dionysiac by introducing new manuscript evidence. He drew attention to the MS Vatican, Biblioteca Apostolica, Vat. Lat. 5755, which preserves a fragment of the Dionysiac *argumenta*, ending with the explicit *Finiunt argumenta paschalium titulorum* after *Argumentum X.*[37] Unfortunately, he did not compare this fragment with Jan's edition, and thus did not comment further on its implications on the corpus of Dionysiac *argumenta.*

Despite this new manuscript evidence, Bruno Krusch decided to republish the entire *Computus Digbaeanus* among Dionysius' computistical works in 1938. His edition, the latest and standard edition to this day, must be regarded as inferior to Jan's for various reasons, among them the fact that he did not compare the text with the *Computus Cottonianus* as Jan had done.[38] Krusch did not discuss the authorship of these *argumenta*, which leaves the reader with the impression that the ascription to Dionysius is valid for all 16,

[36] For this connection see pp. 32, 37, 39–40 below.

[37] Mac Carthy (1901), lvi. For the manuscript see pp. 17–8 below.

[38] Cf. also the verdicts on these editions by Jones (1943), 68–9; Pallarès (1994), 17–8; Wallis (1999), liii.

save for the passages and examples specified as interpolated or accommodated, namely §2 of *Argumentum IV* (with Krusch arguing for a date *c.*582), and the examples given in *Argumenta XI* and *XII*.[39] All this is very surprising considering the fact that Krusch was the first to draw attention to the correspondence of the *Computus paschalis* of 562 attributed to Cassidorus with the first ten Dionysiac *argumenta*, which only led him to an analysis of Cassiodorus' authorship of the *Computus paschalis*, not of Dionysius' authorship of the *argumenta.*[40] Moreover, Lehmann's article on the *Computus paschalis*, published in 1912 and including an edition of that text,[41] must have been known to him and would have had provided him with previously unknown manuscript witnesses.

Manuscript studies of computistical texts were then generally brought to a new level by Charles W. Jones in the late 1930s and early 1940s. Concerning the Dionysiac *argumenta*, he drew special attention to two manuscripts from Mainz (now in the Vatican library: MSS Vatican, Biblioteca Apostolica, Pal. Lat. 1447 and Pal. Lat. 1448), the Vatican manuscript introduced into the discussion by Mac Carthy, and the so-called Sirmond manuscript, i.e. MS Oxford, Bodleian Library, Bodley 309.[42] Unfortunately, he never discussed the transmission of the Dionysiac *argumenta* in these manuscripts in detail, nor did he test his own views on them with van der Hagen's thorough study, which he knew.[43] He simply and persistently argued that only the first nine *argumenta* can be regarded as Dionysiac,[44] which is fairly surprising, since the first ten rather than nine appear as one body in every manuscript he had drawn attention to.[45] An explanation for this opinion is left wanting, and he never commented on possible later interpolations within these nine *argumenta.* Nevertheless, his view was later accepted

[39] Krusch (1938), 62 (where he mistakenly places the mentioning of the consulship of Tiberius Iunior Augustus in *Argumentum VIII* rather than *Argumentum IV*), 76–8.

[40] Krusch (1884), 113–4. For the *Computus paschalis* of 562 see the following note and p. 18 below.

[41] Lehmann (1912), reprinted in Lehmann (1959), with the section on the *Computus paschalis* on pp. 47–55.

[42] Jones (1937), 207, 214; Jones (1943), 69–70. Jones (1943), 69 gives an extensive list of MSS containing Dionysius' computistical writings, stating at the end that the Palatine MSS 'provide valuable evidence about the *argumenta*'. Yet, of the three Palatine MSS listed by him only the two mentioned above contain the *argumenta.* For these MSS see p. 17 below.

[43] Jones (1943), 69.

[44] Jones (1943), 70, 358, 368.

[45] Cf. Jones (1937), 214; Jones (1943), 70, 107 and pp. 16–7 below.

by Wesley Stevens, Peter S. Baker and Michael Lapidge, Dan Mc Carthy, and, as will be seen below, also by Joan Gómez Pallarès.[46] Few statements can be gathered about some of the *argumenta* Jones regarded as pseudo-Dionysiac: For *Argumentum XI* he gave the same reasoning as van der Hagen, arguing that it includes a dating clause for 675, and that the problem of this *argumentum* (the calculation of the epact) had already been dealt with in *Argumentum III*, and 'Dionysius, who is always most concise, would hardly have given two *argumenta* for the same thing; had he done so, he would certainly have given them together';[47] concerning *Argumentum XIII* he argues that it is a 'complex and unsatisfactory' tract originating in the seventh century.[48] Consequently, Jones' criteria for dismissing certain *argumenta* as pseudo-Dionysiac primarily are the late dating clauses on the one hand, the belief in Donysius' competence, clarity and precision on the other. The lack of the latter convinced Jones that *Argumentum XVI* must be pseudo-Dionysiac;[49] concerning *Argumentum XIV* he merely stated that it is probably not Dionysiac, but 'older than Bede'.[50]

In 1958, Alfred Cordoliani compared the *argumenta* attributed to Dionysius with the *Computus paschalis* and what he regarded as the *Computus Cottonianus.*[51] But since the focus of his study was on the *Computus Cottonianus*, he did not comment on the authorship of the other two texts.

Undoubtedly the mathematically most skilled scholar of computistics, Otto Neugebauer, was the first to use the *Computus paschalis* of 562 as evidence for the Dionysiac *argumenta*, in an article published in 1982.[52] In his opinion, the *Computus paschalis* is nothing but a faithful reproduction ('a word-by-word copy') of the original corpus of Dionysiac *argumenta*, in which only the examples were accommodated to 562. Since the *Computus paschalis* comprises *Argumenta I–VI* and *VIII–X* of the 16 *argumenta* in question (and not all of them completely), Neugebauer regarded these as the original corpus, to which he seems to have added *Argumentum XI.* Considering that

[46] Stevens (1981), 90, 107; Baker and Lapidge (1995), xli; Mc Carthy (2003), 37–8; for Pallarès (1994) cf. pp. 13–4.

[47] Jones (1943), 386.

[48] Jones (1943), 355.

[49] Jones (1943), 373.

[50] Jones (1939), 43.

[51] Cordoliani (1958).

[52] Neugebauer (1982), especially 292.

Argumentum XI does not appear in the *Computus paschalis*, that it contains a dating clause for 675, and that the problem described in it was already discussed in *Argumentum III*, the inclusion of this *argumentum* among the original Dionysiac corpus appears rather misguided. Neugebauer's argument here seems to be that *Argumentum XI* deals with 'Easter computus', while *Argumenta VII*, *XII–XVI* do not.[53] This argument does not convince the reader, especially since *Argumentum XIV*, e.g., deals with the calculation of the Easter full moon and Easter Sunday. His other arguments for not including *Argumenta XII–XVI* among the original Dionysiac corpus are textual (different style and different terminology compared to the previous ones).[54] It seems that Neugebauer was not aware of §§2 and 3 of *Argumentum I*, since he worked from Jan's edition of the text, but he quite rightly stated that the chronological data of §2 of *Argumentum III*, which does not appear in the *Computus paschalis*, agrees with the years 522 and 523.[55] Concerning §2 of *Argumentum IV*, which does also not appear in the *Computus paschalis* either, he argued that the mentioned consul Tiberius Iunior Augustus has to be identified with the Byzantine emperor Tiberius, who ruled from 578 to 582. Hence, according to Neugebauer the formula given in this paragraph was applicable to the years counted from 578 onwards, and was probably invented during Tiberius' reign. Yet, the altered formula does not work for years counted from 578, so that Neugebauer's interpretation must be regarded as unlikely.[56] §2 of *Argumenta IX* and *X* do not feature in Neugebauer's discussion.

In an article on Dionysius' dating-system, Gustav Teres, using Jan's edition of the *argumenta* (which he mistakenly ascribes to Benjamin Hoffman), seems not to have studied Jan's notes in detail, since he

[53] Neugebauer (1982), 292, 297, 300.

[54] Neugebauer (1982), 292.

[55] Neugebauer (1982), 293–4.

[56] Neugebauer's argument ((1982), 296) for the modification of the parameter 4 in the formula of §1 of *Argumentum IV* (which represents the *concurrentes* of AD $1-1$) to 1 in §2 is the fact that $577\equiv 3 \bmod 7$. I do not understand the logic of Neugebauer's argument. If he meant that the weekday difference between AD 1 and 578 was 3, then he calculated wrongly, not taking bissextile years into consideration. Anyway, the parameter 1 in §2 of *Argumentum IV* clearly indicates that the *concurrentes* in the first year of the count were 2; moreover, since no parameter is added to the number of years to be divided by four, the first year of the count is supposed to be a year following a bissextile year; both criteria do not agree with 578, the year given by Neugebauer, since it was the second year after a bissextile year, having *concurrentes* 5. Cf. p. 5 above and p. 28 below.

treated the entire corpus without distinction as originally Dionysiac.[57] The only reason for this treatment appears to be the fact that these 16 *argumenta* are published among the works of Dionysius Exiguus, which illustrates the risk an editor takes when publishing dubious material among the original work of an author.

Dáibhí Ó Cróinín, in his 1988 edition of the Irish computistical textbook *De ratione conputandi*, pointed out the fact that this textbook and *Argumentum XVI* discussed similar concepts, which led him to argue that this *argumentum* was 'undoubtedly Insular', 'possibly Irish' in origin.[58]

In 1994, Joan Gómez Pallarès drew particular attention to the *Computus Cottonianus* of 688/9 as an important witness for the Dionysiac *argumenta*, as Jan had done in 1718. This computus (at least as published by Pallarès) contains the first 14 of the 16 *argumenta* in question, none of which preserving the original dating clause of 525. Pallarès edited the section of this computus in which these 14 argumenta appear, comparing their readings with Jan's and Krusch's editions in that process.[59] But this comparison of the *Computus Digbaeanus* (i.e. the *argumenta* as published by Jan and Krusch) with the *Computus Cottonianus* did not lead him to any conclusion about the authorship of the *argumenta* in question. He simply followed Jones in arguing that the first nine are authentic, because they incorporate a dating clause (525) that agrees with Dionysius' other writings; since the same, according to Pallarès, cannot be said about the remaining *argumenta*, one should be careful with attributing these to the Scythian monk.[60] This argument obviously fails to convince, because the example in *Argumentum X* also refers to 525 (though implicitly), and no other reason is given why it should be excluded form the original corpus. Moreover, the differing chronological data in §2 of *Argumenta III* and *IV* are not mentioned.

Arno Borst, in his 1998 book about the Carolingian calendar reform, argued in passing that the *argumenta* incorporating dating clauses for 525 have to be attributed to Dionysius, while he located the addition of §2 of *Argumentum IV* to Italy, dating it to 582, and the addition of arguably the last six *argumenta* to Ireland, dating it to 675.[61]

[57] Teres (1984), especially 182–3.

[58] Walsh and Ó Cróinín (1988), 125, 161.

[59] For the edition see Pallarès (1994), 20–31. Cf. p. 22 below.

[60] Pallarès (1994), 18.

[61] Borst (1998), 177.

In this view he is followed by Kerstin Springsfeld, who more specifically stated that *Argumenta I–X* are Dionysiac because of their dating clauses (save for §2 of *Argumentum IV*, which, according to her, is datable to 582, and probably added to the corpus in Italy), the remaining six, however, pseudo-Dionysiac, presumably composed in Ireland in 675.[62] She possibly saw confirmation for this view in the manuscript evidence, since she was aware of two manuscripts that transmitted *Argumenta I* to *X* as an entity, namely the Sirmond manuscript (MS Oxford, Boldeian Library, Bodley 309) and MS Vatican, Biblioteca Apostolica, Pal. Lat. 1448.[63] Interestingly enough, Springsfeld observed that the chronological data in §2 of *Argumentum III* do not agree with the year 525, but she did not draw the conclusion that this section may not be Dionysiac.[64] Concerning *Argumentum XIV* she argued that it may have been Dionysiac, as the examples given agree with 532–534, basically repeating van der Hagen's observation. The only reason, in her opinion, that could speak against Dionysius' authorship of this *argumentum* is that certain irregularities occur at the end of it; yet, it should be noted that these irregularities only exist in Krusch's edition, not in the manuscript from which h worked.[65] The fact that she connected some of the concepts found in *Argumenta XV* and *XVI* with Irish computistica may have convinced her to ascribed the whole body of the last six *argumenta* to an Irish anonymous.[66]

As mentioned above, Faith Wallis had earlier, in 1999, drawn special attention to the Sirmond manuscript for this question, arguing

[62] Springsfeld (2002), 172–82, the Irish origin of *Argumentum XIV* explicitly on pp. 70, 87, 89, 99.

[63] Cf. Springsfeld (2002), 69, 84. When discussing the Dionysiac *argumenta*, she referred to Jones' table of contents of the Sirmond manuscript (Springsfeld (2002), 172).

[64] Springsfeld (2002), 173.

[65] Springsfeld (2002), 178. Note that her argument here contradicts her statement on pp. 70, 87, 89, 99 that *Argumentum XIV* is of Irish provenance. The two irregularities in *Argumentum XIV* mentioned by Springsfeld are the following: 1) It is argued that the Julian calendar date of the Easter full moon falls on a certain, previously calculated number of days *a Kalendis Januarii*; but it is obvious from the context that the calends of either March or April are meant, depending on the year in question; in fact, *Januarii* is Krusch's own (and wrong) addition, as he clearly indicates in the apparatus. 2) It is argued that if the previously calculated number of days happens to be 30, then the Easter full moon occurs *XXX die Aprilis*; this is obviously an impossible reading, since the Easter full moon falls between 21 March and 18 April; in fact, *Aprilis* is Krusch's correction, as he clearly indicates in the apparatus; the manuscript gives the correct *a Kl* (referring to the calends of March).

[66] Springsfeld (2002), 176.

that the corpus of the first ten *argumenta* found in it has to be regarded as Dionysiac, while disregarding the following *Argumentum XIV* as pseudo-Dionysiac.[67] Unfortunately, Wallis did not discuss these *argumenta* any further, and thus did not mention the chronological and textual problems within these ten *argumenta* as outlined by Jan, van der Hagen, and Neugebauer.

In recent years, the interest in the Dionysiac *argumenta* has led to the reproduction of their text on the world wide web. On one of these sites, Nikolaus Bär provided Krusch's text with a German translation and some commentary.[68] In his *Schlußbemerkung*, he argues that only the first nine *argumenta* formed the original Dionysiac corpus, and among these §2 of *Argumenta III*, *IV* and *IX* represented later additions, without giving further details. He was more specific only in his notes to §2 of *Argumentum IV*, repeating Krusch's argument that this addition must date from the year 582 or later. It seems that his opinion on the original corpus is a more detailed adoption of Jones' view, and thus equally unsatisfactory, especially in terms of *Argumentum X*, whose implicit dating clause for 525 is left unexplained.

The most differentiated treatment of this question since the days of van der Hagen can be found in a brief note by George Declercq in his article about the Dionysiac reckoning, published in 2002.[69] He followed Neugebauer's approach by arguing that the *Computus paschalis* of 562 represents the original corpus of Dionysiac *argumenta* (not adding *Argumentum XI*, as Neugebauer had done). He found confirmation for this view in the fact that every single one of these *argumenta*, and no other among the 16 in question, includes a dating clause for 525. Hence, in Declercq's opinion §2 of both *Argumenta III* and *IV*, *Argumentum VII*, probably §2 of *Argumentum IX*, and *Argumenta XI* to *XVI* are pseudo-Dionysiac. As every scholar before him, however, he seems not to have been aware of §§2 and 3 of *Argumentum I*, being deceived by the editions; at least, he did not include them in his discussion.

Summary

This reassessment of scholarly opinion on the original corpus of the Dionysiac *argumenta* illustrates that the basis of discussion generally was the *Computus Digbaeanus* of 675, the *Computus Cottonianus* of

[67] Wallis (1999), lxxiv.

[68] www.nabkal.de/dionys.html

[69] Declercq (2002), 199.

688/9, and the *Computus paschalis* of 562, while the important manuscript evidence mentioned by Jones was sometimes referred to in passing, but never analysed in detail. Moreover, Jan's and van der Hagen's discussion of Dionysius' authorship, both published in the early eighteenth century, remain the best studies of this question to this day, even though they are based on only a fractional amount of the evidence known to us today. The neglect of these studies during the past century or so led to widely differing comments on the extent of the Dionysiac corpus of *argumenta*, with only Neugebauer and Declercq commenting on this question in some detail. Yet, since both scholars touched this question only in passing and in the end did not agree, neither their accounts, nor the valuable, but outdated discussions of Jan and van der Hagen can be regarded as authoritative in this question. The lacuna of a proper analysis of the extent of the Dionysiac corpus of *argumenta* will be dealt with in the following, since only such an analysis can provide the essential basis of the present study.

Manuscript Evidence for the Original Corpus of the Dionysiac *Argumenta*

As the previous discussion has shown, only five manuscripts containing *argumenta* with the original dating clause of 525, the year in which Dionysius evidently composed his prologue, have been discovered over the past three centuries.[70] For that reason alone, these five manuscripts obviously are the primary witnesses for what may have been the original corpus of the Dionysiac *argumenta.* Since the two existing editions of the *argumenta* are based on only one of these five manuscripts, and none of the remaining four has ever been analysed in detail concerning the extent and structure of the original Dionysiac corpus, these are discussed here first. But it has also become apparent in the preceding discussion that some of the later recensions, most notably the *Computus paschalis* of 562 and the *Computus Cottonianus* of 688/9, also preserve valuable information concerning this question. Yet, these later recensions are numerous; for that reason they will be classified in

[70] The sole manuscript witness for the Dionysiac *argumenta* listed by Cordoliani (1943), 60 is MS Vatican, Biblioteca Apostolica, Reg. Lat. 1260, fols 118–125, which is, in fact, a later recension and does not include the 525 dating. The same holds true for MS Basel, Universitätsbibliothek, F III 15k, mistakenly mentioned by Springsfeld (2002), 73, 76, 84 to include the Dionysiac *argumenta.* Cf. p. 23 below.

terms of their relevance for the transmission of the original Dionysiac corpus of *argumenta* and analysed in these categories. Finally, an overview of this manuscript evidence is provided as the basis for establishing the original structure and subsequent development of the Dionysiac *argumenta*.

MSS Preserving the Original Dating Clause of 525

Group A – MS Oxford, Bodleian Library, Digby 63, 72v–79r (*Computus Digbaeanus* of 675):
This manuscript formed the basis for Jan's and Krusch's edition of the *argumenta*, as well as the present analysis, and is discussed in detail above.[71] Concerning the dating clauses, nine of the 16 *argumenta* transmit the original date 525, namely *Argumenta I–VI* and *VIII* explicitly, *Argumenta IX* and *X* implicitly. §2 of *Argumentum IV*, *Argumentum XIII*, and *Argumentum XIV* incorporate implicit dating clauses for 581, 625, and 532 to 534 respectively, as is discussed elsewhere.[72] The data given in §2 of *Argumentum III* mentions the 10th and 11th year of the *cyclus decemnovennalis* (corresponding to $522+n\times19$), while *Argumenta XI* and *XII* explicitly refer to 675. As mentioned above, this computus has an incipit, but no clearly defined end.[73] The incipit reads: *Incipiunt argumenta de titulis pascalis egiptiorum investigata solercia ut praesentes indicent.*
Group B – MSS Oxford, Bodleian Library, Bodley 309, 81r–82v (Sirmond Manuscript); Vatican, Biblioteca Apostolica, Pal. Lat. 1447, 6v–8v; Vatican, Biblioteca Apostolica, Pal. Lat. 1448, 13r–17v; Vatican, Biblioteca Apostolica, Vat. Lat. 5755, 3–4:
Characteristically, the manuscripts of this group transmit *Argumenta I* to *X* identical in structure and content to the *Computus Digbaeanus* (including §§2 and 3 of *Argumentum I*, which are not included in the main body of the text in the editions), but contrary to that computus these ten argumenta are here strictly defined by incipits and explicits as a single corpus; to this corpus *Argumentum XIV* is attached, in the same version as in the *Computus Digbaeanus.*[74] Hence, the dating

[71] See pp. 3–8 above.

[72] See pp. 8, 20, 28.

[73] Cf. pp. 3–4 above.

[74] The facsimile of MS Vatican, Biblioteca Apostolica, Pal. Lat. 1447, 6v–8v in Appendix III (*Plate* 1) may serve as an illustration of the strictly defined corpus of *Argumenta I–X* with *Argumentum XIV* attached that is the characteristic feature of Group B texts.

clauses in these manuscripts are the exact same as in the *Computus Digbaeanus.*

Jones drew attention to the three manuscripts that contain the full text, while Mac Carthy had earlier referred to a fragment that included only the second half of it.[75] The most important of these manuscripts probably is MS Oxford, Bodleian Library, Bodley 309, which Jones had identified as once in the possession of Jacques Sirmond and studied by the famous early modern scholars of chronology, Petavius and Bucherius, in the seventeenth century, but which was subsequently regarded as lost in the eighteenth and nineteenth centuries.[76] The manuscript itself was written in Vendôme in the eleventh century, but Jones argues that a large part of this manuscript represents the corpus of computistical texts used by Bede for his composition of *De temporum ratione*, among them the *argumenta* in question.[77] Hence, at least in Jones opinion, 'it is definitely possible that O is a copy of a manuscript written before A.D. 725'.[78] If this view is accepted, then this manuscript would contain the oldest known witness of the corpus of *argumenta* attributed to Dionysius. The first ten *argumenta* are clearly defined as one body in this manuscript by the incipit *Incipiunt argumenta Grecorum de titulis paschalibus inuestigata solertia* and the explicit *Expliciunt argumenta paschalium titulorum. Argumentum XIV* occurs as a separate item after that, headed *Incipit calculatio quomodo repperiri posit, quota feria .i. singulis annis .xiiii. luna paschalis, id est circuli decennouenalis* and concluded with the phrase *Haec argumenta hic finiuntur.*

Six years later, in his introduction to Bede's computistical works, Jones then listed manuscripts that contain Dionysius' computistical texts complementary to the manuscripts used by Jan and Krusch. At the end of that list he drew special attention to the Palatine manuscripts in the Vatican as primary witnesses for the study of the Dionysiac *argumenta.*[79] Yet, of the three Palatine codices mentioned in

[75] Cf. pp. 10–11 above.

[76] Jones (1937).

[77] The Dionysiac *argumenta* are item 14 in Jones' table of contents of this manuscript; in his opinion items 13–45 constitute Bede's computus. Jones (1937), especially 214; Jones (1943), 105–10. Cf. Wallis (1999), lxxiv; Springsfeld (2002), 69–70, 76; Ó Cróinín (2003a), 202. For other descriptions of this manuscript see Jones (1939), 126–7; Borst (2001), 159–60; Borst (2006), 263; Mc Carthy and Breen (2003), 27–8; Peden (2003), xl–xli.

[78] Jones (1937), 210. Cf. Ó Cróinín (2003a), 201.

[79] Jones (1943), 69. Cf. also p. 11 above.

his list only two actually contain the *argumenta*, both from Mainz or its vicinity, written in the early ninth century. Of these two, MS Pal. Lat. 1447 is regarded as a copy of MS Pal. Lat. 1448.[80] This is confirmed by the text of the *argumenta*, in which these two manuscripts differ only in form, not in content: The *argumenta* frame an extensive Easter table from 798 to 854 in MS Pal. Lat. 1448, while they are separately copied in MS Pal. Lat. 1447. *Argumenta I* to *X* are clearly defined as one corpus in both codices by the incipit *Incipiunt argumenta Graecorum de titulis paschalibus Aegyptiorum investigata solertia* and the explicit *Expliciunt argumenta paschalium titulorum.* This corpus is then followed by *Argumentum XIV*, headed *Incipit calculatio.*[81]

Earlier, at the turn of the century, Bartholomew Mac Carthy was the first to realise the importance of a fragment that survived in the Vatican MS Vat. Lat. 5755 for the extent of the corpus of Dionysiac *argumenta.*[82] This fragment is famous among scholars of Old Irish, for it contains a considerable number of Old Irish glosses, but it has never been systematically studied by any modern computist.[83] Old Irish scholars tend to argue that the language of the glosses is datable to the eighth century, and that it does not seem likely that these glosses were copied.[84] Therefore, if an eighth century date is assigned to this fragment, it is the oldest surviving manuscript witness of the *argumenta.* The fragment starts on page 3 of the manuscript, which preserves the second half of *Argumentum VIII*, as well as *Argumenta IX* and *X* complete in the same form as in the *Computus Digbaeanus*,

[80] For descriptions of MS Vatican, Biblioteca Apostolica, Pal. Lat. 1447 see Schuba (1992), 257–61 (arguing for Mainz, early saec. IX); Mittler (1986), 128; Lindsay (1915), 481 (Mainz, before 813); Jones (1939), 135 (Mainz, *c.*813); Borst (2001), 73–4; Borst (2006), 298–9 (in both accounts Borst argues for Mainz, between 808 and 813). For descriptions of MS Vatican, Biblioteca Apostolica, Pal. Lat. 1448 see Schuba (1992), 261–5 (Trier and Mainz, saec. IX[1]); Lindsay (1915), 481 (Trier, 810); Jones (1939), 135 (Trier and Mainz, saec. IX[1]); Springsfeld (2002), 84–5 (first part, including the Dionysiac *argumenta*, Trier, 810); Borst (2001), 60–2; Borst (2006), 299–300 (in both accounts Borst argues that the part including the Dionysiac *argumenta* was written in Trier in 810). For MS Pal. Lat. 1447 being a copy of MS Pal. Lat. 1448 see Mittler (1986), 128; Borst (2001), 73.

[81] Cf. Appendix III (*Plate* 1).

[82] Cf. p. 10 above.

[83] For descriptions of this manuscript see Zimmer (1881), xxx; Stokes and Strachan (1901–3), ii xii; Kenney (1966), 671–2; CLA 1, 11 (No. 32). Charles W. Jones, CCSL 123A, xiii lists this manuscripts among computistical texts that 'may have lain within Bede's range', but 'as yet not satisfactorily studied'. This codex also contains a fragment of the *calculus* of Victorius of Aquitaine, cf. Peden (2003), xl.

[84] Stokes and Strachan (1901–3), ii xii; Kenney (1966), 672.

including the implicit dating clauses for 525. *Argumentum X* is then followed by the explicit *Finiunt argumenta paschalium titulorum*, which also marks the end of this page. Page 4 then contains *Argumentum XIV* with the implicit dating clause for 532–534, breaking off after three quarters of the text.[85] According to these details it must be presumed that the manuscipt from which this fragment originates included the first ten Dionysiac *argumenta* as a strictly defined corpus, with *Argumentum XIV* attached, as it appears in the other three codices of this group.

Early Recensions of the Argumenta

Recension A – The *Computus paschalis* of 562:

According to its editor, Paul Lehman, this corpus of *argumenta* survives in five manuscripts.[86] Since it is incorporated in the second book of Cassiodor's *Institutiones* in every single one of these codices, it is assumed that Cassiodor himself may have been the author.[87] Now, by comparing the *Computus paschalis* with the Dionysiac *argumenta* in their published form (i.e. with the *Computus Digbaeanus*), Otto Neugebauer has pointed out that the *Computus paschalis* is nothing but a copy of these *argumenta*, with only two major differences: *Argumenta VII*, *XI–XVI* are missing in the *Computus paschalis* (as they represent *argumenta* that were later added to the original Dionysiac corpus), and the examples in the remaining *Argumenta I–VI*, *VIII–X* were modified to match the year 562 instead of 525.[88] Yet, if the edition

[85] The full text of p. 3 and half of p. 4, including all Old Irish glosses, are edited in Zimmer (1881), 259–61; Stokes and Strachan (1901–3), ii 39–41.

[86] Lehmann (1959), 48; Cordoliani (1943). The five manuscripts are: MS Karlsruhe, Badische Landesbibliothek, Aug. CLXXI, 49v–50v; MS Milano, Biblioteca Ambrosiana, D 17 inf., 52r-v; MS Paris, Bibliothèque Nationale, Lat. 2200, 70v–72v; MS Würzburg, Universitätsbibliothek, M. p. misc. F. 5a, 30v–31v. For the mistaken addition to this list of MS Padua, Biblioteca Antoniana, I 27, 14v–15r in CCSL, Clavis patristica 3A, 269, which stems from this codex being the only one mention under Cassidorus' Computus in Thorndike and Kibre (1963), 1455, see n 112 below.

[87] For the debate concerning Cassidorus' authorship and the incorporation of this corpus of *argumenta* in the second book of the *Institutiones* see Krusch (1884), 113–4; Poole (1918b), 210–1; Lehmann (1959), 47–52; van de Vyver (1931), 289–91; Neugebauer (1982), 292, 301.

[88] For Neugebauer's problematic view that *Argumentum XI* was also part of the original Dionysiac corpus, even though it does not appear in the *Computus paschalis*, see p. 13 above.

of the *Computus paschalis* of 562 is compared with Group A-B texts, further significant textual differences become apparent (some of which are noted by Neugebauer himself[89]): the *Computus paschalis* conflates §§2 and 3 of *Argumentum I*, while §2 of *Argumenta III*, *IV*, *IX*, and *X* do not feature in this text. As regards the incipits and explicits of the *Computus paschalis*, these vary greatly among the five manuscripts, none of which agrees with the ones in Group A-B texts.[90]

Recension B – MS Paris, Bibliothèque Nationale, Nouvelle acquisition 1615, 154r–155r (The *Computus Parisinus* of 819/20):

One important witness for the Dionysiac *argumenta* has escaped the attention of modern scholars. In the ninth century MS Paris, Bibliothèque Nationale, Nouvelles acquisition 1615, 154r–155r from Fleury or Auxerre the Dionysiac *argumenta* are transmitted in a recension of 820.[91] This recension comprises a clearly defined corpus of *argumenta* identical to the corpus of Group B, with *Argumenta VII* and *VIII* being switched. This switch is noteworthy in so far as it demonstrates that the early ninth-century editor of this corpus apparently regarded *Argumenta I–VI* and *VIII* as somehow connected, presumably because of the fact that only these seven include explicit dating clauses.[92] The dating clauses have been accommodated to the year 820 or 819, with few exceptions: No explicit dating clause is given in *Argumentum I*, while the number of indiction cycles that have passed since AD 12 (namely 53) dates this recension to the period between 807 and 821 inclusively; §2 of *Argumentum III* gives the 17th year of the *cyclus decemnovennalis* as example, which agrees with 814; §2 of *Argumentum IV* still preserves the count of years from the year following the consulship of Tiberius Iunior Augustus, which refers to 581;[93] *Argumentum IX* gives data that only matches 813/814 (813: indiction 6 – 7 in MS –, epact 15, Easter Sunday on 27 March, *luna* 20; 814: indiction 7, epact 26, Easter Sunday on 16 April, *luna* 21); *Argumentum X* still preserves the

[89] Neugebauer ((1982), 295–6) notes that §2 of *Argumenta III* and *IV* are not part of the *Computus paschalis.* Cf. also p. 13 above.

[90] For the incipits and explicits of the *Computus paschalis* see Lehmann (1959), 52, 55.

[91] For this manuscript see Krusch (1926), 53 (St.-Benoît-sur-Loire in Fleury, saec. IX); Jones (1939), 130 (Fleury, saec. IX); Borst (2001), 143–5 (Fleury or Auxerre, the latter more likely, *c.*830); Borst (2006), 282–3 (Fleury or Auxerre, *c.*830).

[92] This structure of the *Computus Parisinus* strengthens van der Hagen's view (Hagen (1734), 207) that *Argumenta I–VI* and *VIII* are to be regarded as an entity. For this question see p. 11 above and p. 25 below.

[93] For this dating see p. 28 below.

chronological data of 525 (indiction 3, *concurrentes* 2, Easter Sunday on 30 March). This preservation of the chronological data for 525 proves that the original Dionysiac *argumenta* were the exemplar, and not, e.g., the *Computus paschalis* of 562, while the unaltered inclusion of §2 of *Argumentum IV* (and in fact the entire structure of this *Computus Parisinus*) demonstrates that the examplar was, in fact, a Group B text. These ten *argumenta* are clearly defined as one corpus in the manuscript by a blank line before the first *argumentum* (probably to be later filled by a heading) and by the explicit *Explicit argumenta paschalium titulorum.*[94] In contrast to Group B texts, however, this corpus is not immediately followed by *Argumentum XIV* here, which appears in a different part of the codex (fols 187r-v).

Recension C – Binding fragment of MS Nancy, Bibliothèque municipale, 317 (356) (The *Fragmentum Nanciacense* of 625):

An important manuscript witness for the pseudo-Dionysiac *argumenta*, discovered 140 years ago, has hitherto been overlooked by scholars of early medieval computistics. In 1866, the French palaeographer Henri D'Arbois de Jubainville discovered a binding fragment in the Bibliothèque municipale de Nancy written in an Irish hand of the late eighth or early ninth century;[95] later palaeographers have argued for Bobbio as the provenance of the codex to which this flyleaf is attached.[96] It contains computistical formulae, sporadically glossed in Old Irish. These glosses have made this fragment famous among scholars of Old Irish,[97] but it has never received any attention by modern computists. The recto side of this fragment consists of four passages:[98] 1) The first deals with the length of moonlight per lunar day. 2) The second passage explains how to calculate the lunar age on

[94] Cf. the facsimile in Appendix III (*Plate* 2).

[95] D'Arbois de Jubainville (1866).

[96] The shelf-mark of this codex was 59 when this binding fragment was discovered by D'Arbois de Jubainville (and this number is repeated in all subsequent discussion of the Old Irish glosses found on this flyleaf), but it was changed soon thereafter, in 1873, to 317 (356) (cf. Favier (1886), 123). For a description of this codex see Favier (1886), 176–7; Lindsay (1915), 469; Bischoff (2004), 307; Zimmer (1881), xxx–i; Stokes and Strachan (1901–3), ii xii; Kenney (1966), 672.

[97] The Old Irish glosses are edited in D'Arbois de Jubainville (1886); Zimmer (1881), 262; edited and translated in Stokes and Strachan (1901–3), ii 41; edited, translated and discussed in Gaidoz (1867), 70–1 and D'Arbois de Jubainville (1867). For the Irish context of this fragment see also Zimmer (1881), xxx–i; Stokes and Strachan (1901–3), ii xii; Kenney (1966), 672.

[98] A facsimile of this fragment is reproduced in Appendix III (*Plate* 3) below.

the calends of each month for any given year from fixed regulars for each month[99] and the epact of the year in question; the epact of the example given is 4, i.e. the 15th year of the *cyclus decemnovennalis.* 3) The third passage demonstrates how to calculate the epact from the AD date, which ultimately derives from *Argumentum III*; the example given is AD 701;[100] this calculation is followed by a list of weekday regulars for the calculation of the weekday on the calends of each month for any given year, starting with October,[101] and another list of the lunar regulars for the calculation of the lunar age on the calends of each month, starting with September, which is identical to the list given in the previous passage.[102] This passage is followed by the explicit *Finit argumenta igitur*, which is then followed by the incipit *Alia argumenta nuper inuenta incipiunt amen.* 4) The fourth passage gives formulae for calculating the day of the month and weekday of *luna* 2 of the March lunation from the lunar age of 1 February and the *concurrentes* respectively; this *luna* 2 was significant, because it had the exact same relation to the *initium quadragesimae* (the beginning

[99] These lunar regulars are listed from September to August, and they are identical in order and numerical value to the same list given in the Bobbio Computus 3, 22 (PL 129, 1282, 1289), *Dial. Neustr.* 21 (Borst (2006), 403), *Lect. comp.* I 4 (Borst (2006), 547–8), *Lib. comp.* II 15 (Borst (2006), 1165), Computus of Pacificus of Verona §14 (Meersseman and Adda (1966), 77). Bede lists these regulars from January to December in *De temporum ratione* 20 (Jones (1943), 220–1), as do some manuscripts of *Lect. comp.* and Hrabanus Maurus, *De computo* 70 (CCCM 44, 285), copying Bede. The *Computus Rhenanus* of 775 has both lists (MS Köln, Diözesan- und Dombibliothek, 103, 189v–190r; MS Wolfenbüttel, Herzog-August-Bibliothek, Weißenburg 91, 173r; for this text see pp. 23–4 below).

[100] The AD date is not explicitly mentioned, but for calculation purposes implicitly given as $19\times30+19\times6+17$ (=701); the epact is then correctly calculated as 7.

[101] Usually this list begins with March in computistical texts: So in the Bobbio Computus 3, 21 (PL 129, 1282, 1288); *Dial. Neustr.* 8 (Borst (2006), 389); *Lect. comp.* I 3 (Borst (2006), 546–7); *Lib. comp.* II 15 (Borst (2006), 1165); Computus of Pacificus of Verona §6 (Meersseman and Adda (1966), 75). Bede lists these weekday regulars from January in *De temporum ratione* 21 (Jones (1943), 222), as do some manuscripts of *Lect. comp.* and Hrabanus Maurus, *De computo* 73 (CCCM 44, 289), copying Bede. The *Computus Rhenanus* of 775 has both lists (MS Köln, Diözesan- und Dombibliothek, 103, 189v; MS Wolfenbüttel, Herzog-August-Bibliothek, Weißenburg 91, 173r; for this text see pp. 23–4 below). Interestingly enough, the only computistical text to my knowledge that has the same list of weekday regulars as this fragment, i.e. beginning with October, is the *Computus Cottonianus* of 688/9 (f 75v, cf. n 107 below), while it also appears as a gloss to Willibrord's Easter table (MS Paris, Bibliothèque Nationale, Lat. 10837, 41r) and in the Irish influenced MS München, Bayerische Staatsbibliothek, Clm 14456, 66v.

[102] Cf. n 99.

of the Lenten fast) as the Easter full moon had to Easter Sunday; in fact, this *argumentum* seems to be an adaption of *Argumentum XIV* to the *initium* calculation; the first example given (for this *luna* 2 occurring in February) is the first year of the *cyclus decemnovennalis*, having *concurrentes* 7 (and no provision is made for this year being bissextile), which agrees with AD 703, 798, and 893; since palaeographers argue that the script of this fragment can be dated to the late eighth or early ninth century, 798 seems to be the most likely date for this *argumentum nuper inventum*, and thus the whole fragment may be dated to *c.*798; yet, I would not rule out the possibility of dating this fragment to *c.*703, especially since the previous passage contains a dating clause for 701; of the second example (for this *luna* 2 falling in March) only the first two lines survive.

The verso side of this fragment, then, is the one that concerns us here. It contains only part of the last sentence of *Argumentum XI*, as well as *Argumenta XII* and *XIII* complete. A finit-clause (*Finit amen finit*) appears after *Argumentum XIII*, clearly marking the end of a corpus of *argumenta.* The date given in the examples of *Argumenta XII* and *XIII* is 625, explicitly for the former, implicitly for the latter (the lunar data matches 625/7: 15$^{\text{th}}$ and 17$^{\text{th}}$ year of the *cyclus lunaris*, having epact 16 and 8 respectively).

It is obvious from this analysis that the original manuscript contained at least the end of the second example of the last *argumentum* of the recto side, as well as *Argumentum XI* in the lacuna between the recto and the verso side. Furthermore, it seems plausible to suggest that the incipit before the last *argumentum* on the recto side does not correspond to the explicit on the verso side: The incipit refers to 'recently invented *argumenta*', followed by an *argumentum* with a dating clause for AD 703, 798, or 893, while the *argumenta* on the verso side have dating clauses for AD 625. Therefore I would suggest that another explicit and incipit occurred in the now lost part immediately before *Argumentum XI.* This would imply that *Argumenta XI* to *XIII* were regarded as an entity in this fragment.

The primary importance of the verso side of this fragment is twofold: 1) It preserves the earliest known dating clause for *Argumenta XII* and *XIII*, namely 625; previously, on the basis of the *Computus Digbaeanus*, 675 was regarded as the earliest date at least for *Argumentum XII*, while the few scholars who studied *Argumentum XIII* found it impossible to date it with any certainty; in fact, *Argumentum XIII*

of the *Computus Digbaeanus* preserves the implicit dating clause for 625 given in the *Fragmentum Nanciacense*, which suggests that the author of the *Computus Digbaeanus* worked from an exemplar of *Argumenta XI* to *XIII* of 625 as transmitted in this fragment, accommodating the explicit examples (i.e. the ones explicitly mentioning AD 625 as the precondition), but not *Argumentum XIII* with its implicit data; consequently, it can be safely assumed that *Argumenta XI* to *XIII* are at least as old as 625. 2) Some details of *Argumentum XIII*, especially the mentioning of certain numbers of points, have caused modern scholars some trouble, and they have never been explained satisfactorily; the *Fragmentum Nanciacense* provides a different perspective on these difficulties, which are discussed in detail in Appendix II.

Since this important piece of evidence for the pseudo-Dionysiac *argumenta* has never been transcribed in full, I provide such a transcription in Appendix I.

Recension D: The *Computus Parisinus* of 819/20 is a rare exception of a late recension of the Group B corpus. Generally, the *argumenta* had a different fate from the last quarter of the seventh century onwards: the corpus as preserved in Group B disintegrated, and the argumenta found their way disconnectedly into textbooks, formularies or compendia. Recension D, then, includes every text of whatever form that transmits a later recension of one or more Dionysiac *argumenta* without preserving the Group B corpus.

Since the innumerable texts of this recension obviously do not provide any information about the original corpus of Dionysiac *argumenta*, I will discuss them only briefly here. Such a discussion is nevertheless necessary, since quite a few texts and manuscripts that fall into this category have been connected to the original corpus of the Dionysiac *argumenta* by scholars of computistics, so that their relation to this corpus needs to be clarified. Moreover, the following analysis is designed to illustrate how the *argumenta* were transmitted once the process of disintegration of the corpus had started.

In fact, the *Computus Digbaeanus* marks the first stage in the disintegration of the Dionysiac *argumenta*: Even though the Group B corpus is preserved, the addition of further *argumenta* without clearly marking off the original corpus by means of explicits, incipits or simple headings left the extent of this corpus undefined.

Once the corpus had opened up, it was quite natural that more newly-invented formulae and texts were added, not only to the end of the original corpus, but also into it; moreover, the examples were then usually accommodated to the *annus praesens* of the later compiler. This process, which seems to have started in the last quarter of the seventh century, led to the development of ever growing compilations of computistical texts, in which the original Dionysiac *argumenta* only played a minor role. Yet, at the early stage of this development, the Dionysiac *argumenta* that were included in these compilations usually lost their original dating clause, but remained unaltered otherwise. But once they were clearly disconnected from the Dionysiac corpus, they lost their authoritative attribution, and were then more liable to alteration. Consequently, from the beginning of the eighth-century compilers and authors of computistical textbooks, formularies and compendia started to rephrase the Dionysiac formulae, usually by adding more details and explanations to the calculations, or by generalising them.

The earliest witness to this process of progressive disintegration of the Group B corpus and eventual alteration of the original *argumenta* is the famous *Computus Cottonianus* of 688/9, preserved in one of the few eighth-century computistical codices, namely MS London, British Library, Cotton Caligula A XV, whose provenance appears to be northern France.[103] Pallarès has published the section of this text that includes *Argumenta I* to *XIV*, and compared it with Jan's and Krusch's editions. Yet, the section published by him does not comprise the entire computus, since Pallarès was only interested in the transmission of the *argumenta* ascribed to Dionysius, and thus broke off right in the middle of folio 77r;[104] his choice for breaking

[103] Cf. Lindsay (1915), 461 (arguing that this manuscript was written in France, in 741); CLA 2, 19 (No. 183; Lowe argues that it was 'probably written in North-east France, in a centre with Insular connexions; copied from an exemplar written A.D. 743, the year mentioned on fol. 107'; earlier in this passage he describes the script as 'pre-Carolingian French minuscule, saec. VIII^{2}'); Jones (1939), 120 (northern France, saec. VIII); Pallarès (1994), 20–1. It is the oldest manuscript preserving the Dionysiac *argumenta* in the original or as a recension (a recension in this case); cf. Borst (2006), 398–9.

[104] Pallarès (1994), 20–31. He also defines the *Computus Cottonianus* as comprising fols 73r–77r of this manuscript in two articles of 1987 and 1989, now reprinted in Pallarès (1999), where the relevant references can be found on pp. 26 and 57 respectively (in both cases he mistakenly gives f 73v as the beginning).

off at this point was possibly also influenced by the fact that the following item, i.e. the *Suggestio Bonifati primiceri*, is the only item in this computus that is published separately;[105] yet, this short item, having neither incipit nor explicit that would distinguish it from the rest, seems rather to be an integral part of this computus. In my opinion, folios 73r–80r should be identified as a cohesive formulary, to be termed the *Computus Cottonianus* of 688/9: the beginning of this formulary is clearly defined by the fact that folio 72v is left blank, and thus a new text starts on folio 73r; a later hand adds the heading *Cassiodorus de computo paschali*, an obviously mistaken attribution; the first page after folio 73r, on which the end of an *argumentum* agrees with the end of a page, is folio 80r; no other feature can be found before this page that would mark the end of the text; at the bottom of folio 80r, then, a different hand adds a list of 'canonical' and 'uncanonical' lunar ages, i.e. the 19 epacts on 1 January of the Victorian reckoning and the 11 lunar ages that do not occur on that date in the Victorian 19-year luni-solar cycle respectively; this later addition filling a previously blank space marks the end of this formulary; on fol. 80v, then, starts a recension of the so-called Acts of the Council of Ceasarea.[106]

Now, the *Computus Cottonianus* contains every *argumentum* of the Group B corpus, and at the same time *Argumenta I–XIV*, as well as §2 of *Argumentum XVI* of the 16 *argumenta* of the *Computus Digbaeanus*, most of which are here accommodated to 688/9 and preserved in a structure disrupted by additional *argumenta*: The computus begins with *Argumenta I* to *VIII* (accommodated to 688, and §2 of *Argumenta III* and *IV* omitted), followed by *Argumentum XIV* (including the chronological data for 532 to 534); this is followed by a text on the Julian calendar limits for the Easter New moon, the Easter full moon, and Easter Sunday; then, *Argumenta XI* and *XII* are given (accommodated to 689); attached to *Argumentum XII* (which deals with the calculation of the weekday of 1 January) is a formula to calculate the weekday of the calends of any given month by adding the *concurrentes* of a year to given regulars for each month;[107]

105 This text is edited from this codex and five other manuscripts by Krusch (1926), 55–7.

106 For the Acts of the council of Caesarea see Krusch (1880), 303–10; Jones (1939), 44–5; Wilmart (1933); Strobel (1984), 80–95.

107 This list of regulars, starting with October, is uncommon and has a parallel in the *Fragmentum Nanciacense.* Cf. n 101 above. It is transcribed and discussed in Pallarès (1999), 8–9, who places it in a rather unlikely Arabic context.

this is followed by *Argumenta IX* (with chronological data for 688 and 689), *X* (preserving the chronological data for 525), and *XIII* (preserving the chronological data for 625, and the number of *puncti* as given in the *Computus Digbaeanus*[108]); the remaining part of this computus consists of arguments and texts previously unconnected with the Dionysiac *argumenta*, with the exceptions of §2 of *Argumenta XVI* and *III*, the latter preserving the original chronological data of Group A-B texts.

This description illustrates the degree of disintegration achieved by the *Computus Cottonianus*: the Dionysiac *argumenta* were embedded among numerous other formulae and texts without distinction, while most of the examples were accommodated to the *annus praesens* of the compiler. Consequently, some late seventh and early eighth century computists got to know these *argumenta* through a formulary like the *Computus Cottonianus* rather than through the Group B corpus. In fact, the *Computus Cottonianus* seems to have been quite influential in transmitting Insular computistical knowledge in general, and the Dionysiac algorithms in particular.[109] Its role as one of the earliest Insular sources for Frankish computistics and the extent of its influence have yet to be established, since it plays no part in Arno Borst's source analysis of eighth century Frankish computistical texts. Direct dependency on the *Computus Cottonianus* can be found in a computistical formulary which is preserved in two late eighth- and early ninth-century manuscripts from Cologne and probably Worms, i.e. MS Köln, Diözesan- und Dombibliothek, 103, 184v–190v and MS Wolfenbüttel, Herzog-August-Bibliothek, Weißenburg 91, 169r–173v respectively.[110]

[108] For this problem and the possibility that the *Computus Digbaeanus* and the *Computus Cottonianus* are products from the same computistical school, see Appendix II.

[109] Cordoliani had, quite mistakenly, connected the *Computus Cottonianus* with Spanish computistics: Cordoliani (1942); Cordoliani (1958). This view was corrected by Pallarès (1999), 29–32, 60–2, 99.

[110] For descriptions of MS Köln, Diözesan- und Dombibliothek, 103 see Jaffé and Wattenbach (1874), 40–2; Weber Jones (1932), 32–3; van Euw (1998a) (arguing for Cologne, *c.*795); Bischoff (1998), 397 (probably Cologne, saec. VIII./XI.); Jones (1939), 116 (Cologne, *c.*800); Springsfeld (2002), 82 (Cologne, 810–18); Borst (2001), 63–5; Borst (2006), 238 (Borst argues in both accounts for Cologne, shortly after 810); facsimiles are available on the world wide web: www.ceec.uni-koeln.de. For descriptions of MS Wolfenbüttel, Herzog-August-Bibliothek, Weißenburg 91 see Butzmann (1964), 257–68 (Weißenburg, saec. IX^{1}); Borst (2006), 316–7 (possibly Worms, early saec. IX).

The latest dating clause in this formulary is 775, and thus we may term it the *Computus Rhenanus* of 775.[111] This formulary is an excellent example for the fate of the Dionysiac *argumenta* in computistical compendia of the eighth and later centuries: It copies Bede's version of *Argumentum I*, then gives a version of *Argumentum II* for 775, of *Argumentum VI* for 764, of §2 of *Argumentum III* implicitly for 764 (5th year of the *cyclus decemnovennalis*), of §1 of *Argumentum IV* for 764 and of *Argumentum V* for 764; this is followed by a generalised version of *Argumentum II*, of §1 of *Argumentum III* and *IV*, and of *Argumenta V*, *VI*, and *VIII*; then, among other passages copied from the *Computus Cottonianus*, it transmits the latter's recension (though for 689 instead of 688) of *Argumenta VIII*, *I*, and (obviously without dating clause) *VII*; finally, the first part of *Argumentum XIV* is given. Hence, even though most of the Dionysiac *argumenta* are included in this formulary, none appears in its original form, since the compiler worked from various later recensions of them.

[111] This formulary has not yet been identified as such, and is consequently unpublished. In the descriptions of the Cologne codex, it is denoted as *Canones lunarium decennovalium circulorum* in Jaffé and Wattenbach (1874), 41, and divided into two parts (fols. 184v–187v and fols. 187v–190v) in van Euw (1998a), 134, with reference to the pseudo-Bedan *Canones lunarium decemnovennalium circulorum* (PL 90, 877–82) for the first part. Butzmann (1964), 267 describes this formulary of MS Weißenburg 91 more precisely as consisting to a large degree of the pseudo-Bedan *Canones lunarium decemnovennalium circulorum*, with various insertions. In fact, the entire *Canones lunarium decemnovennalium circulorum* are part of the *Computus Rhenanus*, in which it constitutes *c.* half of the text, and likewise only half of the *argumenta* of the *Computus Cottonianus* that are transmitted in the *Computus Rhenanus* can also be found in the *Canones lunarium decemnovennalium circulorum*. Yet, the relationship between these two texts is rather the other way round, with the *Canones* being a partial copy of the *Computus Rhenanus*: The *Canones* survive in only one manuscript to my knowledge, namely the 10th century MS Köln, Diözesan- und Dombibliothek, 102, 94r–97r (for facsimiles see www.ceec.uni-koeln.de). This codex shows direct dependency on MS Köln, Diözesan- und Dombibliothek, 103, and consequently the *Canones* are copied from the Cologne version of the *Computus Rhenanus*. Jones, in his discussion of the *Canones* in (1939), 82–3, neither refers to the two Cologne codices, nor to the Wolfenbüttel MS. Borst, in his monumental edition of Frankish computistical texts (2006), published *argumenta* of the *Computus Rhenanus* in three different texts, using excerpts from the Cologne version of this computus in his edition of the *Lectiones sive regula conputi* (*Lect. comp.*), *Annalis libellus* (*Lib. ann.*), and *Libri computi* (*Lib. comp.*), and excerpts from the Wolfenbüttel version only in his edition of *Lect. comp.* Accordingly, all four of these texts share common material, but every single one of them deserves a separate treatment, the *Computus Rhenanus* of 775 included.

In general, the textual closeness to the original *argumenta* that is still preserved in the *Computus Cottonianus* was lost in the eighth and early ninth centuries, because the authors and compilers of computistical texts and formulae felt that the *argumenta* either needed more explanations, or should rather be more generalised. Hence, many of the published computistical text from that period include Dionyisac algorithms, but the wording usually varies considerably from the original. Moreover, as has been illustrated in the case of the *Computus Rhenanus* of 775, the formulae were usually copied or altered from an intermediary exemplar, not from the original. Through these intermediary texts the Dionysiac and pseudo-Dionyisac algorithms remained very popular. Some of these can be found in the major computistical works of the eighth and early ninth centuries: Bede's *De temporibus* and *De temporum ratione*, the *Bobbio Computus*, the Frankish texts *Lectiones sive regula conputi* (*Lect. comp.*), *Annalis libellus* (*Lib. ann.*), *Libri computi* (*Lib. comp.*), *Liber calculationis* (*Lib. calc.*), the Computus of Pacificus of Verona, and Hrabanus Maurus' *De computo.*[112] They also appear in considerable variation in MS Basel, Universitätsbibliothek, F III 15k (with examples for 789), which Springsfeld described misleadingly as containing the original Dionysiac *argumenta*,[113] MS Vatican, Biblioteca Apostolica, Reg. Lat. 1260 (with examples for 788), which is the only manuscript listed by Cordoliani for the original Dionysiac *argumenta*,[114] and MS Padua, Biblioteca Antoniana, I 27, which is mistakenly listed as a manuscript witness of the *Computus paschalis*

[112] Bede's *De temporibus* 14 (Jones (1943), 304–5), *De temporum ratione* 47, 49, 52, 54, 57, 58 (Jones (1943), 266, 269, 273–4, 278) ; Bobbio Computus 153–155 (PL 129, 1364–6); *Lect. comp.* IIII 1–7, V 1, VI 6 (Borst (2006), 591–603, 624–6); *Lib. ann.* 2, 3, 6, 8, 15, 20, 24, 25 (Borst (2006), 683–8, 698, 712–3, 715–6); *Lib. comp.* II 7–12b, III 13 (Borst (2006), 1150–6, 1194); *Lib. calc.* 17–23 (Borst (2006), 1397–9); Computus of Pacificus of Verona §§208–9, 213, 216, 219, 249, 250, 309 (Meersseman and Adda (1966), 109–12, 116, 129); Hrabanus Maurus' *De computo* 62, 67, 69, 72, 78, 90 (CCCM 44, 278, 283–5, 288, 295, 312–3).

[113] Springsfeld (2002), 73, 76. Her statement seems ultimately to derive from Jones (1937), 214, who states more precisely: 'The same rubrics with altered formulae in Ba, fo. 37v, from an exemplar written A.D. 789.'

[114] Cordoliani (1943), 60. As so often, Cordoliani trusted the incipit more than the actual content. At least from the dating clauses he should have realised that this is, in fact, a later recension. The incipit reads (MS Vatican, Biblioteca Apostolica, Reg. Lat. 1260, 118r): *Incipiunt argumenta de titulis paschalibus Aegiptiorum inuestigata sollertia, quae Dionisius conposuit utraque lingua Grece uidelicet et Latine eruditus.* This text is somehow related to the *argumenta* in the Basel MS mentioned in the previous note (I have not yet worked out the relationship), which have a very similar incipit.

of 562.[115] Moreover, the computistical excerpts from MS Paris, Bibliohèque Nationale, Nouvelle acquisition 2169 (including a recension of *Argumentum I* from 817) and MS Léon, Biblioteca de la Catedral, N. 8 (including a recension of *Argumentum I* from 806) published by Pallarès fall into this category.[116]

In summary, the texts of this recension do provide valuable information about the later transmission of the Dionysiac algorithms, but only limited insight into what may have constituted the original corpus of Dionysiac *argumenta.* Yet, especially the earliest texts of this recension, which are the most likely to have had the original Dionysiac corpus as their exemplar, show an interesting feature by grouping *Argumenta I–VIII* without their later additions, sometimes excluding *Argumentum VII*: In the *Computus Cottonianus Argumenta I–VIII*, without the later additions to *Argumenta III* and *IV*, appear as one body at the beginning of the text, with all examples being accommodated to 688 (*Argumentum VII* obviously has no dating clause). Bede's *De temporibus* of 703 gives a generalised version of *Argumenta I–VI* and *VIII*, in which only the first paragraph appears in the case of multi-paragraphed *argumenta.*[117] The same holds true for the *Lectiones sive regula conputi* (*Lect. comp.*) of 760, even though §2 of *Argumentum III* occurs slightly later in the text, as it does in the *Computus Cottonianus.*[118] This fact obviously raises the question, whether some seventh and eighth century computists regarded only *Argumenta I–VI* and *VIII* as originally Dionyisac.[119] If they did, the question about the reason and source behind this view remains, since no manuscript is known that

[115] CCSL, Clavis patristica III A, 269 lists MS Padua, Biblioteca Antoniana, I 27, 14v–15r as a witness for the *Computus paschalis* of 562, referring to Thorndike and Kibre (1963), 1455. There, the incipit *Si nosse vis quotus annus est ab* is attributed to the *Computus paschalis.* Yet, this incipit cannot be found on the mentioned folios of the Padua MS, which, in fact, are part of Hrabanus Maurus' *De computo.* It may well be that Thorndike and Kibre actually meant MS Padua, Biblioteca Antoniana, I 27, 86v–87r, with this incipit occurring on f 86v. Yet, the *argumentum* introduced by this incipit continues to calculate 881 as the *annus praesens*; this and the following *argumenta* have no connection to the *Computus paschalis* other than that both texts include later recensions of the Dionysiac *argumenta.*

[116] Pallarès (1999), 67–91.

[117] Bede, *De temporibus* 14 (Jones (1943), 301–2).

[118] *Lect. comp.* IIII 1–7, V 1 (Borst (2006), 591–603).

[119] It should be remembered that van der Hagen (1734), 207, regarded only this group of *argumenta* as certainly belonging to the original Dionysiac corpus. Cf. p. 11 above.

transmits only these 7 *argumenta* with the original dating clause for 525.

Overview

After this discussion of all known manuscripts and texts that contain information about the original corpus of the Dionysiac *argumenta*, it seems appropriate to facilitate the reader with an overview of the details of this analysis, which is done in table form here (*Figure* 1). At the same time, *Figure* 1 will serve as the basis for the following discussion of the extent of the original corpus and the different layers of additions that are part of the *Computus Digbaeanus* of 675.

Defining, Dating and Placing Dionysiac and Pseudo-Dionysiac Argumenta

From the manuscript evidence summarized in *Figure* 1 above the following conclusions can be drawn: Since *Argumenta XI–XIII* only appear in the *Computus Digbaeanus* (Group A) and the *Fragmentum Nanciacense* (Recension C) and incorporate seventh-century dating clauses, they can safely be regarded as pseudo-Dionysiac; the same is valid for *Argumenta XV* and *XVI*, which have no dating clauses, but appear only in the *Computus Digbaeanus* of the texts listed above. Consequently, *Argumentum XIV* is embedded among pseudo-Dionysiac *argumenta* in that formulary; moreover, it appears separated from the main corpus of *argumenta* by an explicit and incipit in Group B, and does not appear in any of the recensions.[120] For these reasons it seems that it should also be classified as pseudo-Dionysiac, but the implicit dating clause for 532 to 534 suggests that it needs more analysis. Of the first ten *argumenta*, only the parts that appear in all groups and recensions and include a dating clause for 525, the year in which Dionysius evidently wrote his prologue, can, at this stage, be safely considered as Dionysiac, namely §1 of *Argumentum I*, *Argumentum II*, §1 of *Argumenta III* and *IV*, *Argumenta V*, *VI*, *VIII*, and §1 of *Argumenta IX* and *X*. Yet, doubt remains concerning the last mentioned, since they are not included in the recensions of these *argumenta* in

[120] Note that in the Paris MS, *Argumentum XIV* is physically disconnected from Recension B (*Computus Parisinus* of 819/20). Cf. p. 19 above.

Argumentum	I			II	III		IV		V	VI	VII	VIII	IX		X	
§	1	2	3		1	2	1	2					1	2	1	2
Group A	525	x	x	525	525	10, 11 of cd (579)	525	(581)	525	525	x	525	(525/6)	x	(525)	x
Group B	525	x	x	525	525	10, 11 of cd (579)	525	(581)	525	525	x	525	(525/6)	x	(525)	x
Recension A	562	y		562	562		562		562	562		562	(562)		(562)	
Recension B	post 806 pre 822	z	z	820	820	17 of	819 cd (814)	(581)	819	819	VIII 819	VII	(813/4)	x	(525)	x
Recension C																

Argumentum	**XI**	**XII**	**XIII**	**XIV**	**XV**	**XVI**
§						
Group A	675	675	(625/7)	(532)	x	x
Group B				(532)		
Recension A						
Recension B						
Recension C	partially	625	(625/7)			

Figure 1 The extent and chronological details of the Dionysiac and pseudo-Dionysiac *argumenta* according to their primary witnesses.

AD dates explicitly mentioned in the *argumenta* stand alone, while implicit dates are given in brackets. If an *argumentum* contains no dating clause, x stands for its inclusion in the respective corpus of *argumenta*, y and z for its inclusion in variation, while a blank space indicates that it is not included. '10, 11 of cd' denotes the 10th and 11th year of the *cyclus decemnovennalis*, which represents the chronological data given in that *argumentum.* Roman numerals indicate that the *argumentum* with that number appears in that place in the respective corpus.

Bede's *De temporibus* and the *Lectiones sive regula conputi* (*Lect. comp.*), and they appear disconnected from the first eight *argumenta* in the *Computus Cottonianus.* Thus, all the paragraphs and *argumenta*, which cannot readily be attributed to Dionysius, as well as the evidently pseudo-Dionysiac *argumenta* need to be analysed in detail concerning their most likely date and place of composition and their addition to the Dionysiac corpus. It is apparent from the manuscript evidence that whichever parts of the first 10 *argumenta* were later added to the original corpus, these additions happened before the corpus as defined in Group B and Recension B (*Computus Parisinus* of 819/20) found a wider distribution, and hence before further *argumenta* were attached to this strictly defined corpus. Therefore, the controversial passages among the first ten *argumenta* are analysed and summarised first, followed then by *Argumenta XI* to *XVI.*

§§2 and 3 of *Argumentum I*: The first editor of the Dionysiac *argumenta*, Wilhelm Jan, did not include these two paragraphs in his edition, even though they appear in his sole manuscript witness, while the second editor, Bruno Krusch, relegated them to the apparatus.[121] Thus, both editors did not regard them as genuinely Dionysiac. This is an odd treatment of these paragraphs considering that both editors had no scruples in publishing *argumenta* with seventh-century dating clauses in the same corpus. Anyway, the reason for discarding these paragraphs seems to have been their content: as outlined above,[122] the formula given in §1 needed to be modified every 15 years. The need for this modification is explained in general terms in §2, including the example of the next modification, i.e. the increase of the multiplicand by 15 from 34 to 35, which would become necessary in 537. §3, then, expands this example to the subsequent modification, i.e. the increase of the multiplicand by 15 from 35 to 36, which would become necessary in 552. Moreover, §3 gives more mathematical details about the application of the algorithm in years of such a modification, in this case the year 552: instead of adding indiction 15 in this year, the multiplicand by 15 is increased by one; in subsequent years, however, the indiction will then have to be added again until indiction 15 is reached; at this point the multiplicand by 15 will again have to be increased by one, from 36 to 37, which would become necessary in 567. Consequently, the argument for discarding these paragraphs as pseudo-Dionysiac seems to be that they deal with problems that would not

[121] Jan (1718), 81; Krusch (1938), 77.

[122] Cf. p. 4 above.

need to concern Dionysius at his time of writing. Additionally, these detailed further explanations seem out of place considering the short and concise descriptions of the following formulae.

Yet the formula given in §1 could only be fully understood and applied in subsequent years with at least the additional explanations outlined in §2. Moreover, the parameter calculated in *Argumentum I*, i.e. the AD date, is the sole precondition of the following six *argumenta* (*II–VI* and *VIII*); consequently, the correct calculation of this parameter was vital, and the algorithm was not complete without the explanation of the modifications that become necessary every 15 years. This need for further details alone explains the different structure of *Argumentum I* compared to the plain formulae of the subsequent ones. If §§2 and 3 are, however, considered together, the discussion of the modifications to the algorithm seems indeed unnecessarily extensive compared to the very brief and precise style of the subsequent *argumenta.* Moreover, every detail concerning the modifications to the algorithm was readily and precisely explained in §2; §3 only supplies further nuances, which are illustrated by the example of the year 552. On these textual grounds it seems to me that only §2 was part of the original corpus of Dionysiac *argumenta*, while §3 was added in or just before the year 552. This view is supported by the manuscripts evidence: while both paragraphs occur in Groups A and B and in adapted form in Recension B, §3 does not appear in Recension A, i.e. the *Computus paschalis* of 562, which is a crucial recension for this analysis of the shape of the original Dionysiac corpus. In consequence, this implies that a paragraph was added to the original corpus as early as *c.*552, while the author of the recension of 562 worked from the original, pre-*c.*552 corpus; but it seems also possible that this third paragraph was added at a later stage, with its author regarding an extension of the example of §2 as the best way of providing more details for the modification of the formula given in §1.

§2 of *Argumentum III*: There are various conclusive reasons for dismissing this paragraph as pseudo-Dionisiac: it does not appear in Recension A; it explicitly states that it was invented later (*Item alium computum nuper inuentum*); the calculation differs from the general structure of calculating from the AD date; the chronological data refers to the 10th and 11th year of the *cyclus decemnovennalis*, which is incompatible with 525, the date given in §1 of this *argumentum.* The data corresponds, in fact, with $522+n\times19$.[123] Since this paragraph

[123] Cf. pp. 5, 13 above.

appears in Groups A and B, as well as Recension B, it seems plausible to suggest that it was inserted at this place before the *argumenta* received a wider circulation, i.e. before the end of first quarter of the 7th century. The 10th year of the *cyclus decemnovennalis* occurred in the following years in the period between 525 and 625: 541, 560, 579, 598, and 617. Of these, none is really close to the date suggested for the addition of §3 of *Argumentum I*, i.e. ca. 552. In fact, the similarity in style of the first sentence of this paragraph (cited above) with the first sentence of §2 of *Argumentum IV*, which can be dated to 581 or shortly thereafter, seems to suggest that the paragraph in question here was composed close to this date. Hence, I regard 579 as the most likely date for its invention, and that it was then added to the Dionysiac corpus in or slightly later than 581.

§2 of *Argumentum IV*: That this paragraph is a later addition is explicitly expressed in its first sentence: It was invented later for the purpose of simplifying the calculation of the preceding paragraph (*Item nuper inventa melius iudicavi, si brevius patefiat*). This simplification was achieved by counting the years not from Christ's incarnation, but from the first year after (*post*) the consulship of Tiberius Iunior Augustus; because of this change in the precondition, the fixed parameter given in the algorithm had to be modified accordingly from 4 to 1. Since the number of years was counted inclusively, this fixed parameter denotes the *concurrentes* of the first year of the count minus one. Moreover, the count of years in the formula of §1 obviously starts with the first year in a four-year bissextile cycle, i.e. a year following a bissextile year (namely AD 1), since no parameter is added to the number of years to be divided by four, i.e. every fourth year of the count was bissextile; as no further modifications in this respect are mentioned in §2, the same must be valid for the first year of the new count.[124] Hence, the first year after the consulship of Tiberius was supposed to have *concurrentes* 2 and to follow a bissextile year. Various suggestions have been made concerning the AD date of the year in question, but these suggestions were almost never accompanied by any further explanation.[125] There

[124] For the details of the algorithm cf. p. 5 above.

[125] A late hand in the margin of MS Oxford, Bodleian Library, Digby 63, 74r notes 'AD 581', which was adopted by Jan (1718), 84; the present analysis proves that this is the correct date. Van der Hagen (1734), 207 regards 703 as the date in question. Krusch (1938), 62, 76 argues for 582, and is followed in this view by Borst (1998), 177; Springsfeld (2002), 172; Bär, http://www.nabkal.de/dionys.html. Neugebauer (1982), 296, makes a case for 577. Cf. pp. 9–15 above.

can be hardly any doubt that the Tiberius referred to in this passage is the Byzantine Emperor Tiberius Constantinus. According to the generally accepted chronology, Tiberius ruled from 578 to 582.[126] Yet, of these years only 581 agrees with the above mentioned chronological data, and thus this must have been the year regarded as the first year after Tiberius' reign by the author of this paragraph. Consequently, this paragraph seems to have been composed in 581 or shortly thereafter, presumably in a place in or with close connections to the Byzantine Empire; since it was apparently added before this corpus got a wider distribution, some place in Italy with the just mentioned attribution seems to be most likely.[127]

Argumentum VII: It should be remembered that Dionysius compiled the *argumenta* to provide algorithms for calculating the data of the columns of his Easter table.[128] From that perspective, *Argumentum VII* seems well placed here, since it deals with the Easter full moons, which are listed in the column following the years of the *cyclus lunaris*, the topic of *Argumentum VI.* Yet, van der Hagen doubted that this *argumentum* had been part of the original Dionysiac corpus for the reason that it has a structure different from the other *argumenta*, as it provides no formula, but only a list.[129] More conclusive than this argument is the manuscript evidence: *Argumentum VII* does not occur in Recension A and in a different place in Recension B. Consequently, this *argumentum* appears to be a later addition, which was presumably inserted into the corpus in or after 552, i.e. the possible date for the earliest addition (§3 of *Argumentum I*), but before it got a wider circulation in the early seventh century. It seems rather unlikely that it was added at the same time as §2 of *Argumenta III* and *IV*, since the phrasing is different, as is the quality of the *argumentum* (formula vs list). As will be discussed in the following, it may have been added at the same time as §2 of *Argumentum IX.*

Argumentum IX: As mentioned earlier, two aspects suggest that this and the following *argumentum* may not be Dionysiac: 1) In some later transmissions of the Dionysiac algorithms *Argumenta I–VIII*

[126] For Tiberius and the period of his reign see Schreiner in LM 8, 760–1 (*s.v.* Tiberios).

[127] Borst (1998), 177; Springsfeld (2002), 172 also argues for Italy. Cf. p. 14 above.

[128] See the citation given on p. 2 above.

[129] Van der Hagen (1734), 207.

appear as a well-defined group, while these two *argumenta* either do not appear at all, or are transmitted disconnected from this group;[130] 2) the structure of these two *argumenta* is strikingly different to the previous ones, since they do not have the AD date as their precondition. On the other hand, there are many conclusive arguments that *Argumentum IX* was part of the original Dionysiac corpus: it supplies an algorithm to calculate the lunar age of Easter Sunday, which constitutes one of the columns in the Dionysiac Easter table, and which was, at the same time, the most debated question in the Easter controversy; the chronological data of the two examples given in this *argumentum* agree with 525 and 526;[131] *Argumentum IX* is included in all Groups and Recensions, especially also in Recension A (again in considerable variation);[132] most conclusive of all, *Argumentum IX* is referred to and applied in the *Suggestio* of Bonifatius, the chancellor of the papal curia, to pope John I, written in 526.[133] From this evidence there cannot be any doubt that this *argumentum* was certainly part of the original Dionysiac corpus.

Yet the question remains whether this holds true for all parts of this *argumentum.* The reason for outlining two examples instead of just one is that different fixed parameters had to be applied, depending on whether Easter Sunday fell in April or March.[134] Thus, both examples were necessary to understand the algorithm, and for that reason alone both were quite certainly part of the original *argumentum.* Serious doubts exist, however, concerning the second paragraph of this *argumentum.* Scholars have generally avoided a discussion of this paragraph, and if they commented on it, opinions on the meaning of this paragraph vary, and it is yet to be explained convincingly.[135] The key to an understanding of this paragraph may lie in an interpretation of *Argumentum IX* to be found in the *Lectiones sive regula conputi* (*Lect. comp.*): chapter VI 6 of that text consists of a generalised version of §1

[130] Cf. p. 25 above.

[131] Cf. p. 7 above.

[132] Cf. p. 25 above.

[133] Krusch (1926), 56–7.

[134] Cf. p. 7 above.

[135] Comments on this paragraph can only be found on the world wide web: Deckers (http://hbar.phys.msu.su/gorm/chrono/paschata.htm) suggests that this paragraph deals with the calculation of the lunar age of any given day of January, while Bär (http://www.nabkal.de/dionys.html) rather thinks that this paragraph is supposed to point out that the algorithm of §1 of this *argumentum* can also be applied for calculating the lunar age of any given day of a year.

of *Argumentum IX*, followed by a slightly altered version of §2 of that *argumentum*, which is then followed by the interpretation in question.[136] The editor of this text, Arno Borst, lists five manuscripts for this interpretation, none of which is older than the middle of the ninth century, while the dating clause of the oldest version suggests that it was first composed in 792.[137] According to this interpretation, §2 of *Argumentum IX* explains nothing but a different composition of the fixed parameters in the algorithm of §1.[138] Only the fixed parameter for April is discussed: if 3 is added to the number of months from September to December (4 inclusively), then the sum is 7; if the last two days of December are added to this, then the sum is 9; this is the fixed parameter applied for calculating the lunar age of Easter Sunday, if the latter happens to fall in April. It is very striking that a discussion of the fixed parameter for March is left wanting in this interpretation, which is the most confusing part of §2 of *Argumentum IX.* In that part of §2 it is argued that in bissextile years 2 instead of 3 has to be added to the number of months from September to December (4 inclusively). But a bissextile year obviously makes no difference to the lunar calculation explained in §1 of this *argumentum.* If the early ninth century interpretation reproduces the original meaning of §2, then 'bissextile years' has rather to be read as 'years in which Easter Sunday falls in March'; the reading would then basically be that the March regular consist of the number of months between September to December

[136] *Lect. comp.* VI 6 (Borst (2006), 624–6).

[137] *Lect. comp.* VI 6D (Borst (2006), 626). The five manuscripts are: MS Bern, Burgerbibliothek, 417 (Fleury, 826); MS Karlsruhe, Landesbibliothek, Augiensis CLXVII (Soissons/ Laon, *c.*848); MS Köln, Diözesan- und Dombibliothek, 83^2 (Cologne, 805); MS London, British Library, Harley 3017 (Fleury, *c.*863); MS St. Gallen, Stiftsbibliothek, 248 (northern France, *c.*850). Borst seems also to include MS Ge, but since no variants are given for this MS, I suspect that it should rather have been listed among the manuscripts in which this passage does not occur. All manuscripts give the dating clause for 792 (epact 23, Easter Sunday on 15 April, *luna* 17) except for the Bern MS, in which the dating clause is accommodated to 825 (epact 28, Easter Sunday on 9 April, *luna* 16).

[138] This interpretation is supported by the fact that different explanations for the fixed parameters in this algorithm were composed as early as 526: Boniface, the chancellor of the papal curia in his *Suggestio* to pope John I of 526, explained the fixed parameter 9 for April as consisting of the number of months from September to April divided by 2 (8/2=4) plus the five days that the Egyptian year of 360 days is shorter than the Julian calendar year of 365 (Krusch (1926), 56–7). Hence, §2 of *Argumentum IX* was probably just one of a number of different explanations for the fixed parameters in this algorithm, but obviously the only one that made its way into the Dionysiac corpus of *argumenta.*

(4 inclusively) + a fixed parameter of 2 + the last two days of December, adding up to 8. The author of §2 may have regarded the first example given in §1 of this *argumentum* as a bissextile year, which led him to the assumption that the fixed parameter for March had to be applied in every bissextile year. Whatever the case may be, the mentioning of the bissextile year is totally misplaced in this context (since it makes a difference in weekday, but not in lunar calculations), and it appears that the compiler of this paragraph did not fully understand the preceding examples. Consequently, it seems very likely that §2 of *Argumentum IX* represents a later, confused addition, which is confirmed by the fact that it does not appear in Recension A. As for a possible date for the addition of this paragraph, it obviously found its way into the Dionysiac corpus before the emended corpus as represented in Group B and Recension B got a wider distribution, i.e. before the end of the first quarter of the seventh century. This paragraph cannot really be connected with §2 of *Argumenta III* and *IV*, since their precise chronological argument is not matched by the confused explanation here. Moreover, the purpose of this paragraph was not to add a new or simpler formula, but to provide additional explanation to the existing argument. Hence, if this paragraph was added at the same time as any of the other additions, I would tend to connect it with *Argumentum VII*, but not with §3 of *Argumentum I* and §2 of *Argumentum X*, which are more technical, and very precise. In the end, however, the most likely scenario seems to be that it was added separately.

Argumentum X: As has been mentioned in the discussion of *Argumentum IX*, *Argumentum X* does not appear in some later transmissions of the Dionysiac *argumenta*, and its structure is strikingly different compared to the first seven *argumenta* (*I–VI, VIII*), which casts doubts on its inclusion in the original corpus. Additionally, *Argumentum X* deals with weekday calculation, and thus is not immediately connected to any column in the Dionysiac Easter table. Yet, the example given here is to calculate the weekday of Easter Sunday of 525,[139] and thus this *argumentum* may have been included in the corpus as a method for double-checking the solar Easter data, and thus would have had an immediate application for the Easter table (it could obviously also be used for calculating the weekday of the Easter full moon). Moreover, the implicit date in this *argumentum* suggests that it was part of the original corpus, and more decisively it figures

[139] Cf. p. 7 above.

in Groups and Recensions A and B discussed above. However, §2 of this *argumentum* does not feature in Recension A. In this paragraph it is argued that the weekday of any given Julian calendar date from 1 January to 31 December can be calculated with the algorithm of §1; this algorithm is here outlined again, this time not by means of an example, but in general terms. Such a generalisation of the preceding formula is not given for any of the other *argumenta*, and it does not fit into Dionysius' general scheme of explaining the parameters of his Easter table. For these reasons, as well as the fact that this paragraph is not transmitted in Recension A, I believe that this paragraph also represents a later addition, which may be connected to §3 of *Argumentum I*, as will be argued in the following; at least it was added to the Dionysiac corpus before that got a wider distribution in form of the Group B corpus.

Argumenta I–X: In summary, of the 10 *argumenta* preserved in the Group B corpus, §§1 and 2 of *Argumentum I*, *Argumentum II*, §1 of *Argumenta III* and *IV*, *Argumenta V*, *VI*, and *VIII*, and §1 of *Argumenta IX* and *X* can be regarded as Dionysiac and seem to have formed the original corpus of Dionysiac *argumenta.* On the other hand, §3 of *Argumentum I*, §2 of *Argumenta III* and *IIII*, *Argumentum VII*, and §2 of *Argumenta IX* and *X* have been identified as pseudo-Dionysiac. But can these additions be classified, i.e. were some of these added at the same time? Only §2 of *Argumenta III* and *IV* can be connected with some degree of certainty, since both have very similar phrasing in their first sentence, and they are the only additions that provide new formulae. Because of these formulae, §2 of *Argumentum III* can be dated implicitly to 579, §2 of *Argumentum IV* to 581 or shortly thereafter. Hence, it seems that these two paragraphs were added to the corpus in 581 or shortly thereafter. As for the place of this addition, the reference to the Byzantine emperor Tiberius Constantinus in §2 of *Argumentum IV* suggests a place that was either part of the Byzantine empire, or at least in close contact with it. We need not to assume that the original corpus had travelled very far at this early stage, so some place in Italy, either under Byzantine rule or with strong Byzantine connections, seems most likely. As for the remaining four additions, §3 of *Argumentum I* and §2 of *Argumentum X* have a quality distinctly different from the other two, since they provide further detailed technical explanations from the hand of a skilled computist, while *Argumentum VII* is a mere list, which could have been compiled

from any Easter table, and §2 of *Argumentum IX* is a confused explanation of the fixed parameters of the preceding algorithm. Thus, if these four additions were to be grouped, I would be inclined to connect §3 of *Argumentum I* with §2 of *Argumentum X*, and *Argumentum VII* with §2 of *Argumentum IX*. §3 of *Argumentum I* suggests the date 552 for its addition, but it may well have been added retrospectively; I would not rule out the possibility that §3 of *Argumentum I* and §2 of *Argumentum X* were added at the same time as §2 of *Argumenta III* and *IV*. *Argumentum VII* and §2 of *Argumentum IX*, however, seem to me to be disconnected from the other four additions.

All this must obviously remain very speculative. Nevertheless, for the outlined reasons I would at least propose that §2 of *Argumenta III* and *IV* were added to the corpus in 581 or shortly thereafter, §3 of *Argumentum I* and §2 of *Argumentum X* earlier or at the same time, while *Argumentum VII* and §2 of *Argumentum IX* were presumably added later, but before the end of the first quarter of the seventh century; all these additions seem to have been included before the corpus, as it is now preserved in Group B and Recension B, left Italy.

The complexity of the different levels of the just discussed first ten *argumenta* has been noted in passing by only very few scholars, most notably Jan and van der Hagen, and, more recently, Neugebauer and Declercq. Concerning the subsequent *Argumenta XI* to *XVI* in the *Computus Digbaeanus*, however, recent scholars like Borst and Springsfeld believe that they were possibly invented, but certainly added to the corpus in 675, presumably in Ireland.[140] This view needs serious revision, as the following discussion will show.

Argumenta XI to *XIII*: The opinion that *Argumenta XI* to *XVI* were possibly composed and added to the corpus in 675 is based on the fact that *Argumenta XI* and *XII* in the *Computus Digbaeanus* contain dating clauses for that year, which have been the earliest dating clauses for these *argumenta* known to modern scholars of computistics. Yet, an important piece of evidence, discovered 140 years ago, has been overlooked by these scholars, namely the *Fragmentum Nanciacense* discussed above.[141] This fragment suggests that the three *Argumenta XI* to *XIII* formed an entity, and that they

[140] Cf. pp. 9–15 above.
[141] Cf. pp. 19–21 above.

were probably invented in 625, if not earlier. 625 would have been a very appropriate date for the composition of new *argumenta*, since the Dionysiac Easter table ended in the following year, 626, and the continuation of that Easter table made re-calculations necessary.[142]

Having established 625 as the most likely date for the composition of *Argumenta XI* to *XIII*, the question of their provenance remains. This early date obviously suggests that they were composed somewhere in Italy or Spain, because these were the only Latin speaking places where the Dionysiac reckoning was accepted at that time. Rome, the Frankish kingdoms, Christian Kent and Northumbria would rather have followed the Victorian reckoning, while the 84 (14)-year Easter reckoning still prevailed in Ireland and the British kingdoms, as well as Pictland.[143] In fact, two facts speak strongly for the western Mediterranean world in general, and Italy in particular: 1) The provenance of the manuscript to which the *Fragmentum Nanciacense* is attached is believed to be Bobbio, and the same may be true for this flyleaf;[144] 2) in *Argumentum XIII* computistical methods are applied, which are also described by Maximus Confessor in his *Computus ecclesiasticus* of 641, and which may ultimately go back to fifth century north African computistics.[145] Maximus wrote his computitistical treatise in the North African part of the Byzantine Empire and therefore this *argumentum* may also have been composed in a place under Byzantine control, or at least with close connections to the Empire, somewhere between Bobbio and North Africa.

There is no evidence that this group of *Argumenta XI–XIII* was attached to the Dionysiac corpus (or rather the Group B corpus)

[142] For the end of the Dionysiac Easter table see Krusch (1938), 74. The necessary re-calculations are explained by Dionysius himself in his prologue (Krusch (1938), 64). Since *Argumenta XII* and *XIII* deal with the calculation of the solar and lunar data for 1 January, it seems to me that these *argumenta* were invented to add columns for 1 January to the continuation of the Dionysiac Easter table. Columns for the *feria* and *luna* of 1 January can be found in later Easter tables of the Dionysiac reckoning, e.g. in an Easter table for 798–854 transmitted in MS Vatican, Biblioteca Apostolica, Pal. Lat. 1447, 19v–22r; MS Vatican, Biblioteca Apostolica, Pal. Lat. 1448, 13v–16r; MS Vatican, Biblioteca Apostolica, Reg. Lat. 1260, 112r–114v.

[143] Unfortunately, there is no authoritative study on the adoption of the different Easter reckonings in Western Europe. Valuable, but often outdated information on this question can be found in: Krusch (1884); Poole (1918a–b); Schmid (1904); Schmid (1907); Jones (1934).

[144] Cf. n 96 above.

[145] Cf. Appendix II below.

before it was included in the *Computus Digbaeanus* in 675; consequently it seems to me that this group of *Argumenta XI–XIII* and the Group B corpus had disconnected histories before 675.

Argumentum XIV: This *argumentum* is one of the most popular in early computistical literature. The reason for this popularity obviously is its concise method for calculating the Julian calendar date and the weekday of the Easter full moon, as well as the Julian calendar date and lunar age of Easter Sunday. Its earliest version provides an implicit dating clause for 532 to 534, and it is this version which accompanied the Dionysiac *argumenta* in Group B and which was then incorporated in the *Computus Digbaeanus* and the *Computus Cottonianus.*[146] The chronological data were later adapted to match the years 684–686, which was probably done very close to these years.[147] Another and more detailed version of this *argumentum* was composed in 776 and is commonly ascribed to Alcuin.[148] Moreover, numerous variations of his *argumentum* can be found in later computistical texts and manuscripts.

Even though the implicit dating clause of the earliest version suggests 532 as the date of composition, there are good reasons to doubt this. If it was composed in 532, one would expect it to be quite naturally included in the corpus of Dionysiac *argumenta.* Yet, it is not part of, nor does it travel with Recension A of 562, and neither is it connected with Recension B; moreover, even though this *argumentum* is attached to Group B texts, it is at the same time markedly separated from the main corpus of *argumenta.* Consequently, there is no evidence that it became part of a larger corpus of *argumenta* before its inclusion in the *Computus Digbaeanus* and the *Computus Cottonianus.* Thus, from the existing evidence the incorporation of *Argumentum XIV* into a larger corpus can be dated to 675.

From the manuscript evidence it seems, then, that *Argumentum XIV* was composed and attached to the Dionysiac *argumenta* after the inclusion of all additions to its main body, i.e. after the Group B corpus was established, but before this corpus got a wider distribution, since it is

[146] This version is discussed in detail on pp. 8–9 above.

[147] This version can be found, e.g., in MS Paris, Bibliothèque Nationale, Lat. 4860, 151r-v (cf. Springsfeld (2002), 181); MS München, Bayerische Staatsbibliothek, Clm 14456, 46v–47v. The characteristic of this version is that the *concurrentes* given in the three examples are 5, 6, and 7 respectively.

[148] For this version see especially Jones (1939), 43–4, 104–6; Springsfeld (2002), 183–5, 322–8.

attached to every Group B text. Hence, I would suggest the late sixth or early seventh century for its composition, before the Group B corpus with *Argumentum XIV* attached left Italy. This is confirmed by the fact that the algorithm described in *Argumentum XIV* was at least partially known to Maximus Confessor in 641;[149] it is possible that Maximus worked directly from *Argumentum XIV*; on the other hand, it is at least as likely that the algorithm itself was an old invention of the Greek speaking East (with Alexandria as its computistical centre), but that it became known to the Latin speaking West only by the late sixth or early seventh century through *Argumentum XIV.* Either way, this would again suggest that *Argumentum XIV* was composed (in Latin) in a centre somewhere in Italy that was part of, or at least in contact with the Byzantine Empire. The examples for 532 to 534 in this *argumentum* were then obviously chosen because they represent the first three years in the Dionysiac Easter table, which were regarded as appropriate, neutral examples.

Argumenta XV and *XVI*: It has been suggested so far that none of the previous *argumenta* was actually composed in 675. Consequently, only *Argumentum XV* and *XVI* may have been original contribution by the author of the *Computus Digbaeanus*, and therefore only these two *argumenta* may provide a clue about the provenance of this computus. They are not *argumenta* in van der Hagen's strict sense, since they do not supply mathematical algorithms; for that reason they can also not be dated with certainty. From the evidence of the *Computus Digbaeanus*, it seems, however, quite certain that these *argumenta* were composed either in 675 or earlier. *Argumentum XV* deals with theological explanations of the equinoxes and solstices, as well as the chronology of Christ's life. *Argumentum XVI* discusses the bissextile day, including a curious division of the 8760 hours of a year by 7 to explain the annual increase of the bissextile day. Now, the topic of *Argumentum XV* was prominent in Bede's *De temporum ratione*, as well as in Irish computistical textbooks of the late seventh and early eighth century, and from these sources it found its way into Frankish computistics.[150] The division

[149] Cf. Maximus Confessor, *Computus ecclesiasticus* I 19 (PG 19, 1235–8).

[150] Cf. Bede, *De temporum ratione* 30 (Jones (1943), 236); Munich Computus (MS München, Bayerische Staatsbibliothek, 8r–46r; an edition is forthcoming) f 19r; *Computus Einsidlensis* (MS Einsiedeln, Stiftsbibliothek, 321 (647), 82–125) pp. 98–9; Bobbio Computus 29, 45 (PL 129, 1291, 1297); *Lib. comp.* I 3b (Borst (2006), 1104); *Lib. calc.* 82 (Borst (2006), 1433). For the recently discovered *Computus Einsidlensis* see Warntjes (2005) and now also Bisagni and Warntjes (2008). For the theological explanations of the equinoxes and solstices discussed in this *argumentum* see also Strobel (1977), 290–303.

given in *Argumentum XVI*, however, curious as it is, cannot be found in Bede's computistical writings, but was particularly popular in Irish computistical textbooks contemporary to the *Computus Digbaeanus*, and then later in Frankish computistics, drawing on these Irish sources.[151] From this perspective it seems that one has to consider Ireland rather than Anglo-Saxon England or the continent for the provenance of the *Computus Digbaeanus.*

On the other hand, it is a very curious fact that only the dating clauses of *Argumenta XI* and *XII* were accommodated to the *annus praesens* of 675 in the *Computus Digbaeanus.* The reason for this may have been that the compiler knew about the significance of the 525 date, and did not want to undermine Dionysius' authority by changing the dating clause. Only *Argumenta XI* and *XII*, then, explicitly referred to a year other than 525, namely 625, and the compiler saw no reason why he should not accommodate this to his *annus praesens.* Yet, he seems not to have been computistically skilled enough to change the implicit dating clause of *Argumentum XIII*, which still preserves the 625 dating. Anyway, the fact that the compiler of the *Computus Digbaeanus* actually worked on two of these *argumenta* is interesting in itself. None of the Irish computistical textbooks mentions any of the algorithms given in the first 14 *argumenta*; the calculations in these textbooks are executed by strikingly different and simpler methods. In fact, the term *annus domini* itself is not mentioned in any of them, and neither is the term *concurrentes*, both being absolute essentials for an understanding of the Dionysiac formulae. From the evidence of these textbooks it seems that Irish computists applied methods of calculation to the Dionysiac reckoning that they had already used for the *latercus* (the 84 (14)-year reckoning) and the Victorian reckoning; these had proven their value over decades, if not centuries, and Irish computists felt no need to adopt formulae based on new chronological concepts they were unfamiliar with, like *annus domini.* For these reasons I suspect that the *Computus Digbaeanus* was compiled in an Anglo-Saxon centre rather than an Irish one, considering especially that Bede used the Dionysiac *argumenta* as early as 703.[152] Yet,

[151] Munich Computus fols 22v–23r; *Computus Einsidlensis* p. 108 (for this passage cf. Bisagni and Warntjes (2008), 89–90); Bobbio Computus 39, 40 (PL 129, 1295–6); *Dial. Burg.* 14 (Borst (2006), 366); *De Bissexto* (PL 101, 994–5; for this text see especially Springsfeld (2002), 203–14); *Lib. comp.* I 3f (Borst (2006), 1106).

[152] Bede, *De temporibus* 14 (Jones (1943), 301–2). For further arguments for an Anglo-Saxon authorship of the *Computus Digbaeanus* see Appendix II.

as mentioned above, the division given in *Argumentum XVI* does not appear in Anglo-Saxon, but is prominent in Irish computistica. Therefore, my best guess is that the *Computus Digbaeanus* was compiled in an Anglo-Saxon centre in Ireland, such as Rath Melsigi, but monastics centres in Anglo-Saxon England certainly cannot be ruled out.[153]

Conclusion

This article has demonstrated that the text published as the *argumenta* attributed to Dionysius, i.e. the *Computus Digbaeanus* of 675, comprises not only the original Dionysiac corpus, but also different layers of additions, which illustrate (if not represent) the development of computistical formularies written in Latin from 525 to 675. The different stages in this development are the following:

1) The original Dionysiac corpus of 525 is the first known computistical formulary written in Latin. It has not survived in its original extent in any manuscript, but can be reconstructed, primarily on the basis of Recension A (the *Computus paschalis* of 562), as consisting of the following *argumenta*: §§1 and 2 of *Argumentum I*, *Argumentum II*, §1 of *Argumenta III* and *IV*, *Argumenta V*, *VI*, and *VIII*, §1 of *Argumenta IX* and *X*.

2) In the course of the sixth and possibly early seventh century, §3 of *Argumentum I*, §2 of *Argumenta III* and *IV*, *Argumentum VII*,

[153] The provenance of the manuscript (which is highly disputed; cf. n 16 above) is irrelevant for the question of the provenance of the *Computus Digbaeanus*, since a text could have travelled anywhere in the 200 years between the composition of the text and that of the codex. For Anglo-Saxon centres in Ireland in general, and Rath Melsigi in particular, see especially Ó Cróinín (1984), reprinted in Ó Cróinín (2003), 145–65. It should be noted that the hypothesis of the *Computus Digbaeanus* being composed in an Anglo-Saxon centre in Ireland depends heavily on the evidence of *Argumentum XVI*, which, in the end, may not even have been part of *Computus Digbaeanus* proper (cf. p. ?? above); additionally, even though the curious mathematical explanation of the annual bissextile increase outlined in *Argumentum XVI* cannot be found in Bedan computistica, it is, interestingly enough, attributed to a certain Theodore in the *Computus Einsidlensis* p. 108; if the identification of this Theodore with Theodore of Tharsus, archbishop of Canterbury (as proposed by Bisagni and Warntjes (2008), 89–90) is accepted, then the *Computus Digbaeanus* may as likely have been composed in Anglo-Saxon England in general, in Theodore's Canterbury school in particular.

§2 of *Argumenta IX* and *X* were added to the original corpus. Only the addition of §2 of *Argumenta III* and *IV* can be dated with some certainty to 581 or slightly later; it has been suggested that §3 of *Argumentum I* and §2 of *Argumentum X* were presumably added at the same time or earlier (possibly 552), while *Argumentum VII* and §2 of *Argumentum IX* may have been added later. All these additions seem to have been composed and integrated in the original corpus before that corpus got a wider distribution outside of Italy; §2 of *Argumenta III* and *IV* probably originated in a centre under Byzantine control, or at least with some connection to the Empire. This body of *argumenta* is transmitted as a strictly defined corpus in Group B texts and Recension B (the *Computus Parisinus* of 819/20).

3) *Argumentum XIV* was then attached to this Group B corpus, presumably in the late sixth or early seventh century, in all likelihood also in Italy, possibly in a Byzantine centre, or a centre with contact to the Empire. Yet, this *argumentum* was kept strictly separated from the main corpus, as witnessed in Group B.

4) *Argumenta XI* to *XIII* were in all likelihood composed as a group in 625 (as illustrated in Recension C, i.e. the *Fragmentum Nanciacense*), in Bobbio or some other Italian centre with Byzantine connections. There is no evidence that this group was connected with or even attached to the Group B corpus before its inclusion in the *Computus Digbaeanus*, but it is not unlikely that it left Italy in a manuscript which also included the Group B corpus.

5) Finally, the group of *Argumenta XI* to *XIII* was inserted between the Group B corpus and *Argumentum XIV*, and *Argumenta XV* and *XVI* were added to this enlarged corpus; additionally, the marked separation of these groups was abandoned. This newly defined corpus of 16 *argumenta* survived uniquely as the *Computus Digbaeanus* of 675, which was probably compiled in an Anglo-Saxon centre, possibly rather in Ireland (such as Rath Melsigi) than in Anglo-Saxon England proper, though the latter option can by no means be ruled out.

The abandonment of the strict definition of the Group B corpus in 675 basically opened this corpus up to further additions, which ultimately led to its complete disintegration. The earliest witness to this

disintegration is the *Computus Cottonianus* of 688/9. Even though the phrasing of the *argumenta* remained essentially unaltered in this text, almost all of the original dating clauses were already accommodated to the *annus praesens* of the compiler. Not long after that, in 703, Bede began the process of generalising these argumenta, in which only the algorithms remain unchanged. In the eighth and ninth century, then, the Dionysiac formulae were usually only known through later recensions.[154]

Appendices

Appendix I: Argumenta XI–XIII in the Fragmentum Nanciacense (Nancy, Bibliothèque Municipale, 317 (356))

<Argumentum XI>

. . . <rem>anent VII: septima luna in XI Kalendas Aprilis . . .

<Argumentum XII>

Si uis nosse diem Kalendarum Ianuarium per singulos annos quota sit feria, sume annos ab incarnatione Domini nostri Ihesu Christi, ut put<a> annos DCXXV; deduc assem, remanent DCXXIIII. Hos per quartam partem partire, et quartam, quam partitus es, adiecies super DC<XXIIII>, fiunt DCCLXXX. Hos partiris per septem, remanent III; tertia feri<a> Kalendarum die Ianuarii. Si IIII, quarta feria; si V, quinta feria; si VI, sexta <fe>ria; si asse, dominicus; si nihil, sabbatum.

<Argumentum XIII>

Si uis scire, quota sit luna Kalendis Ianuariis, scito quotus lunaris ciclus lunaris XV est. Tene tibi unum, id est, ipsam diem Kalendas Ia<nu>arias, et deduces quinques deciis, V quinques, faciunt LXXV; quos <ad>iecias super unum, et fiunt LXXVI. Item ducis sexies deciis, V sexies, faciunt LXL; quos adiecias super LXXVI, et fit numerorum summa CLX<VI>, in quibus partiris tricissima, remanent XVI; sexdecima luna est in Kalendis Ianuariis, et puncta XLII. Isto modo per X et VIIII ciclos lunares conputa<bis> semper in Kalendis Ianuariis, ut quota sit luna absque errore re<pperi>es.

Dum autem ueneris ad XVII ciclum lunarem et duxeris quinques deciis <se>ptus super Kalendas Ianuarias, qui faciunt dies LXXXV.[155]

[154] For a brief sketch of the fate of the Dionysiac formulae from the late seventh to the early ninth century see pp. 21–5 above.

[155] Here follows mistakenly: Deinde d<u>cis sexies, octuagenta v.

Si partiris sexagissima et adiecias ipsum as<sem>, fiunt LXXXVI. Deinde duces sexies deciis septus, fiunt CII; quos adiecias <super> LXXXVI, et fiunt CLXXXVIII. Partiris ibi tricissima, remanent VIII; octaua luna est in Kalendis Ianuariis et puncta XXIIII. Sic et in XVIII et XVIIII ciclo facies. A primo uero ciclo lunari usque in XVII non partiris sexaginsimam partem ne in errorem incedas. Finit Amen fini<t>.

Appendix II: Explanation of Argumentum XIII

Argumentum XIII has puzzled modern computists over the past three hundred years and has never been satisfactorily explained in every detail. Jan admitted that he did not entirely understand *Argumenta XI* to *XVI*,[156] and this was arguably especially true for *Argumentum XIII.* It seems that the number of points mentioned in this *argmentum* irritated him more than anything else; since he did not know how to interpret these numbers, he referred to Bede (*De temporum ratione* 24), who described the increase and decrease of moonlight by four points per day (with five points defining an hour), and to *Argumentum XVI*, where an hour is defined as consisting of four points; he then left it to the reader to establish, which of these two theories was to be applied to make sense of the points mentioned in this *argumentum*, if any sense could be made of them. Additionally, Jan notes various variants (especially in numbers) from the *Computus Cottonianus*, illustrating that even medieval copyists were confused by this text. Van der Hagen, then, drew attention to the fact that this *argumentum* has parallels with the *Computus ecclesiasticus* of Maximus Confessor. At the same time, he pointed out that the algorithm of *Argumentum XIII* can be found in *De temporum ratione* 57, but that Bede does not mention any superfluous points.[157] It is through this chapter in *De temporum ratione* that the formula given in *Argumentum XIII* is generally well explained,[158] and I have described the general method in detail above.[159] This

[156] Jan (1718), 88.

[157] Van der Hagen (1734), 208.

[158] Cf. Jones (1943), 278, 388–9; Wallis (1999), 141 (both argue somewhat misleadingly that the lunar age of 1 January is 1 in the first year of the *cyclus lunaris*; correct is rather that the fixed parameter 1 in the formula refers to the lunar age of 1 January of the preceding, i.e. the 19th and final year (cf. p. 8 above); they are confused by the fact that Bede started the *cyclus lunaris* a year early in his description of it in *De temporum ratione* 56; for this inconsistency in Bede's account see also Pillonel-Wyrsch (2004), 344–5).

[159] Cf. p. 8 above.

formula became quite popular in eighth- and early ninth-century computistics, since it can also be found in the *Bobbio Computus, Lectiones sive regula conputi (Lect. comp.), Annalis libellus (Lib. ann.), Libri computi (Lib. comp.), Liber calculationis (Lib. calc.),* and Hrabanus Maurus' *De Computo.*[160]

Nevertheless, the formula is not as clearly and plausibly explained in *Argumentum XIII* as it is in these other computistical texts (with the exception of *Lect. comp.*), which is primarily due to the fact that different computistical techniques are used in *Argumentum XIII*; moreover, some additional details are mentioned in *Argumentum XIII*, like the superfluous points, that cannot be explained by any of the other versions of the formula. How confusing the account in *Argumentum XIII* appears to the modern computist is illustrated by the fact that Springsfeld could not make sense of *Argumentum XIII*, even though she explains the same algorithm in *Lib. ann.* perfectly.[161] Still, good mathematical discussions of this *argumentum* by Nikolaus A. Bär and Michael Deckers can now be found on the world wide web.[162] Yet, Bär does not discuss the problem of the meaning of the points, while Deckers cautiously suggests that they may refer to the annual increase of the *saltus lunae.*

In general, there are three aspects that make this *argumentum* difficult to understand:

1) The first is the partition of the multiplicand 11 into 5 and 6 (i.e. 11=5+6). This may slightly confuse the reader, but it obviously makes no difference in the calculation. Such a partition is very uncommon in early medieval computistics, since it does not really simplify the algorithm. Van der Hagen had drawn attention to the fact that Maximus Confessor referred to people who followed the same practice;[163] yet, for their calculations this

[160] Bobbio Computus prefix (PL 129, 1281); *Lect. comp.* V 5 (Borst (2006), 607; Borst argues that this chapter deals with the calculation of the lunar age of any given Julian calendar day; yet, the algorithm given rather refers to the calculation of the lunar age of 1 January; interestingly enough, the multiplicand 11 is here also divided into 5 and 6, as in *Argumentum XIII*); *Lib. ann.* 42 (Borst (2006), 738); *Lib. comp.* IIII 3 (Borst (2006), 1201); *Lib. calc.* 31 (Borst (2006), 1402) ; Hrabanus Maurus, *De Computo* 79 (CCCM 44, 296).

[161] Springsfeld (2002), 175–6, 356–7.

[162] Nikolaus A. Bär: http://www.nabkal.de/dionys.html; Michael Deckers: http://hbar.phys.msu.su/gorm/chrono/paschata.htm.

[163] Maximus Confessor, *Computus ecclesiasticus* I 11, 12, II 1, 5 (PG 19, 1227–30, 1251–4, 1261–2).

partition of 11 into 5 and 6 was a necessity.[164] Since the formulae described by Maximus Confessor are different from the one in *Argumentum XIII*, but similar in their method of calculation, it may be inferred here that the author of *Argumentum XIII* came from a similar, if not the same computistical school as the computists referred to by Maximus Confessor. Since Maximus Confessor wrote his treatise in North Africa, and since the computistical methods described by him ultimately go back to fifth-century Northern African computistics,[165] the author of *Argumentum XIII* possibly also came from that region; at least it is hardly likely that this *argumentum* was composed outside the western Mediterranean world.[166]

2) More problematic is the second example given in *Argumentum XIII.* After explaining the algorithm by means of the 15th year of the *cyclus lunaris*, which undoubtedly refers to the *annus praesens* of the composition of this *argumentum*, i.e. AD 625,[167] the author does not choose the following year as his second example, but rather the 17th of the *cyclus lunaris.* The reason for this is that for the 17th to the 19th year an alteration of the formula became necessary, which the author wanted to illustrate here: the *saltus lunae* is applied in the 16th year, which led to an increase of 12 instead of 11 lunar days between the 16th and 17th year; consequently, 1 had to be added to the existing formula.[168] Now, this addition of 1 was executed in the most unlikely manner in this *argumentum*, namely by arguing that the product of the year in the *cyclus lunaris* multiplied by 5 has to be divided by 60 and the integer from that division has then to be added to the product. This division by 60 was to be applied only for years 17 to 19 of the *cyclus lunaris.* Now, since the products of 17, 18 and 19 multiplied by 5 range between 60 and 120 ($17\times5=85$, $18\times5=90$, $19\times5=95$), the integer when divided by 60 is 1 in all three cases. This odd division by 60 is a reminiscence of calculations described by Maximus Confessor, in which

[164] For these calculations see especially Schwartz (1905), 82–4. Maximus Confessor himself (*Computus ecclesiasticus* I 27 (PG 19, 1245–6)) calculated the lunar age of 1 January from the year in the *cyclus decemnovennalis* without partitioning the multiplicand 11.

[165] See especially Schwartz (1905), 70.

[166] For this question see also p. 32 above.

[167] For this *annus praesens* see p. 32 above.

[168] For this alteration of the algorithm see also p. 8 above.

5/60=1/12 denoted the increase of the *saltus* per year, i.e. the 12-year *saltus* of the *Supputatio Romana*, and again points to North African computistics.[169] Because of this association of the division by 60 with the *saltus lunae*, the addition of one that resulted from the *saltus lunae* being applied in the 16th year of the *cyclus lunaris* is described in these odd terms here.

Unfortunately, this second example is not transmitted uncorrupted in the three primary witnesses of *Argumentum XIII*, i.e. the *Fragmentum Nanciacense*, the *Computus Digbaeanus* and the *Computus Cottonianus.* The problem is that in all three accounts the fixed parameter 1 in the algorithm, which was introduced in the first example, is not mentioned in this second example: The *Fragmentum Nanciacense* calculates consistently without this fixed parameter, thus establishing the lunar age 8 for 1 January of the 17th year of the *cyclus lunaris*; in the *Computus Digbaeanus* and the *Computus Cottonianus* this parameter is silently added: in the *Computus Digbaeanus* 102+86=189, in the *Computus Cottonianus* 85+1=87, eventually resulting in *luna* 9 on 1 January of the 17th year of the *cyclus lunaris.*

How, then, did this corruption occur? There cannot be any doubt that the original *Argumentum XIII* mentioned this fixed parameter 1, since the second example was designed to demonstrate the impact of the *saltus lunae*, i.e. the extra addition of 1, on this calculation; the formula would *de facto* not have been altered, if this fixed parameter 1 was omitted. Consequently, *Argumentum XIII* must have been tampered with later, and for a specific reason: In the original *Argumentum XIII* the *saltus lunae* was applied before 1 January of the 17th year of the *cyclus lunaris*, presumably at the end of the previous November lunation; yet, a different tradition also existed, namely to apply the *saltus* at the end of the following March lunation, i.e. after 1 January of the 17th year of the *cyclus lunaris.* Hence, I suspect that followers of this March place of the *saltus* altered this example for calculating the lunar age of the 17th year of the *cylus lunaris* by omitting the fixed parameter 1, and thus arrived at *luna* 8, which they regarded

[169] See especially Maximus Confessor, *Computus ecclesiasticus* I 12, II 5 (PG 19, 1227–30, 1261–2) and Schwartz (1905), 82–3.

as correct, rather than *luna* 9. This altered version of the original *Argumentum XIII* is transmitted through the *Fragmentum Nanciacense*, and this alteration may well have been executed in Bobbio.[170] It was such an altered version, then, that received a wider distribution throughout Western Europe, while it was later silently corrected by followers of the November place of the *saltus*, presumably by Anglo-Saxon computists, as illustrated in the *Computus Digbaeanus* and *Computus Cottonianus.*[171]

3) Yet the aspect that puzzled modern commentators on *Argumentum XIII* most is the mentioning of the points. First of all it needs to be pointed out that the *Computus Digbaeanus* and the *Computus Cottonianus* transmit identical numbers, which disagree with the numbers of points given in the *Fragmentum Nanciacense.* As will be presently seen, this is due to the fact that different concepts are are uunderlying these two versions of *Argumentum XIII.* I will deal with both in turn:

a) The *Computus Digbaeanus* and the *Computus Cottonianus* give i) *luna* 16 and 16 points on 1 January of the 15th year of the *cyclus lunaris* and ii) *luna* 9 and 26 points on 1 January of the 17th year of the *cyclus lunaris.* What is meant by these points here is the exact time of the day at which the moonrise occurs, i.e. the number of points between the beginning of the first hour of that day and moonrise. The algorithm is explained in Frankish computistics, namely in the *Computus Rhenanus* of 775, *Lectiones sive regula conputi (Lect. comp.), Annalis libellus (Lib. ann.), Libri computi (Lib. comp.), Liber*

[170] In later computistics, the March place of the *saltus lunae* was a specifically Irish application, while the November place became standard in Anglo-Saxon computistics. This is not only evident from Irish and Bede's computistical textbooks (as well as Alcuin's letters), but it is also explicitly expressed by the Irish computist Dicuil, writing in the Frankish kingdoms in 814–816 (Dicuil, *Liber de astronomia* I 5 (Esposito (1907), reprinted in Esposito (1990), 338): *Etsi lunarem saltum in vigesimo quarto die mensis Novembris, secundum Anglos, complere volueris* [. . .]. *Sed si secundum Grecorum ac Latinorum regulam, quam mea gens in Hibernia in hac ratione semper custodit, praedictum saltum in vigesimo secundo die mensis sequentis, iuxta primum tempus creationis lunae rationabiliter observaveris.* The Irish may well have adopted a practice that was earlier followed in some Italian centres with Irish connections, most notably Bobbio.

[171] For the association of the place of the *saltus lunae* in November with Anglo-Saxon computistics see the previous note. For further arguments for an Anglo-Saxon authorship of the *Computus Digbaeanus* see p. 34 above.

calculationis (*Lib. calc.*):[172] Count the number of days from 1 January to the day in question; add the product of the year of the *cyclus lunaris* in which this day occurs multiplied by 5; divide this sum by 60; then the remainder denotes the number of points between the beginning of the first hour and moonrise; in mathematical terms:

(number of days from 1 January+year in *cyclus lunaris*×5)mod60 = points between beginning of first hour and moonrise

If this theory is applied to the examples in *Argumentum XIII*, then the results are exactly the numbers of points mentioned:

i) the number of days from 1 January to 1 January is 1 (counted inclusively); the year of the *cyclus lunaris* is 15, and 15×5=75; hence, (1+75)mod60=16, i.e. the number of points given in *Argumentum XIII*; since 5 points equal 1 hour in this algorithm, this means that on 1 January in the 15th year of the *cyclus lunaris* the moon rises in the first point of the fourth hour of the day.

ii) the number of days from 1 January to 1 January is 1 (counted inclusively); the year of the *cyclus lunaris* is 17, and 17×5=85; hence, (1+85)mod60=26, i.e. the number of points given in *Argumentum XIII*; since 5 points equal 1 hour, this means that on 1 January in the 17th year of the *cyclus lunaris* the moon rises in the first point of the sixth hour of the day.

Again, this algorithm shows parallels to the calculations described by Maximus Confessor, even though the applications are different.[173] Nevertheless, the feature of multiplying the year of the *cyclus lunaris* by 5 and dividing the product by 60 seems in both cases to be a reminiscence of the 12-year *saltus* of the *Supputation Romana*, since 5/60=1/12. The general idea of this division seems to

[172] *Computus Rhenanus* (MS Köln, Diözesan- und Dombibliothek, 103, 185v; MS Wolfenbüttel, Herzog-August-Bibliothek, Weißenburg 91, 170r; for this text see pp. 23–4 above); *Lect. comp.* V 9 (Borst (2006), 609); *Lib. ann.* 40 (Borst (2006), 736); *Lib. comp.* IIII 2 (Borst (2006), 1201); *Lib. calc.* 32 (Borst (2006), 1403). Borst's commentary to these chapters is misleading.

[173] Cf. Maximus Confessor, *Computus ecclesiasticus* I 12, II 5 (PG 19, col. 1227–30, 1261–2) and Schwartz (1905), 82–3.

have been to calculate the impact of the *saltus lunae* on the lunar calculation of any given year of the *cyclus lunaris*, even though the 12-year *saltus* had obviously no connection to a 19-year luni-solar cycle. The wrong application of mechanisms used for the *Supputatio Romana* to a 19-year luni-solar cycle, and thus the close resemblance with calculations described by Maximus Confessor (ultimately deriving from fifth-century North African computistics) appears to suggest that this algorithm either originated in North Africa in Maximus Confessor's time or earlier, or that methods introduced by North African computists later found their application in the West.

b) On the other hand, the equivalent passage in the *Fragmentum Nanciacense* reads '*luna* 16 and 42 points' and '*luna* 8 and 24 points'. These numbers of points can be explained by an uncommon system for calculating the length of moonlight per lunar day. The common theory of this question in the early Middle Ages is that the length of moonlight increases and decreases with the waxing and the waning of the moon by 4 points per lunar day: the length of moonlight for *luna* 1 is 4 points, for *luna* 2 8 points, etc. until it is 60 points for *luna* 15; from *luna* 15 it then decreases by 4 points, so that it is 56 points for *luna* 16, etc. until it is 4 points for *luna* 29; 5 of these points constitute an hour, so that the longest period of moonlight is 12 hours on *luna* 15. This theory can be found in the *Bobbio Computus*, *De temporum ratione*, *Annalis libellus* (*Lib. ann.*), *Libri computi* (*Lib. comp.*), the Computus of Pacificus of Verona, and Hrabanus Maurus' *De computo.*[174] The theory applied in the *Fragmentum Nanciacense* version of *Argumentum XIII* is slightly different in that it can be established that the increase and decrease of the length of moonlight is 3 points per lunar day rather than 4: for *luna* 8: $3\times8=24$ points, for *luna* 16: $15\times3-1\times3=42$ points. In this theory it seems that an hour is

[174] Bobbio Computus 65 (PL 129, 1305); Bede, *De temporum ratione* 24 (Jones (1943), 226–7); *Lib. ann.* 44, 45 (Borst (2006), 740–1); *Lib. comp.* IIII 5 (Borst (2006), 1202);, Computus of Pacificus of Verona §§337–342 (Meersseman and Adda (1966), 136–7); Hrabanus Maurus, *De computo* 43 (CCCM 44, 255–6). For this theory cf. Jones (1943), 359; Springsfeld (2002), 358; Pillonel-Wyrsch (2004), 196–200 (including a rather meaningless comparison with modern observations).

defined as consisting of 4 points rather than 5, so that the longest period of moonlight, i.e. for *luna* 15, would be 45 points=11¼ hours, which can be compared to the 12 hours given in the common theory. Now, this interpretation of the numbers of points in the *Fragmentum Nanciacense* would have to remain speculative, were it not for the fortunate fact that the theory described is transmitted in a computistical MS, namely the often cited mid-eighth century (and therefore one of the oldest computistical codices that have survived) MS London, British Library, Cotton Caligula A 15, 106v:

Incipit de luna quantum lucit in noctae
Momenta XII faciunt uncia una, et uncias III faciunt puncto I, et puncti quattuor faciunt ora una.

Luna prima lucet uncias VIIII, qui faciunt punctis III; luna secunda hora et dimidia lucet; luna tertia horas II et uncias III; luna quarta horas III lucet; luna quinta horas III et uncias VIIII; luna sexta horas IIII et uncias sex; luna septima horas V et uncias III; luna octaua horas sex; luna nona horas sex et uncias VIIII; luna X horas septem et uncias sex; luna XI horas VIII et uncias III; luna XII horas nouem; luna XIII horas VIIII et uncias VIIII; luna XIIII horas X et uncias sex; luna XV horas XI et uncias III; luna XVI horas XI; luna XVII horas X et uncias III; luna XVIII horas X et uncias VI; luna XVIIII horas VIIII et uncias VIIII; luna XX horas nouem; luna XXI horas VIII et uncias III; luna XXII horas VII et uncias VI; luna XXIII horas VI et uncias VIIII; luna XXIIII horas sex; luna XXV horas V et uncias tres; luna XXVI horas IIII et uncias VI; luna XXVII horas III et uncias VIIII; luna XXVIII horas tres; luna XXVIIII horas II et uncias III; luna XXX hora una.

Note that this passage agrees with the theory outlined above only in its first half, namely by recording a consistent increase of moonlight by 3 points per lunar day from *luna* 1 to *luna* 15. In the second half, however, this passage is inconsistent by recording a decrease of only one point from *luna* 15 to 16, an increase (!) of a point from *luna* 17 to *luna* 18, and a decrease of 5 points from *luna* 29 to *luna* 30. It may well be that the copyist was only familiar with the common theory, realised when copying the data for *luna* 15 that the theory he copied does not agree with what he was

> used to (11 ¼ instead of 12 hours of moonlight for *luna* 15), and then tampered with the following data to get closer to his own data.

A few words remain to be said about the transmission of *Argumentum XIII.* As has been pointed out above, it seems that the original *argumentum* (α) has not survived, since the mentioning of the fixed parameter 1 was deliberately omitted in the second example (I denote this version, in which the fixed parameter was omitted in the second example, as β). Moreover, since two different concepts for the calculation of the mentioned points are applied in the surviving recensions of this *argumentum*, the question remains which of these concepts was applied in α. Since the methods of calculation used in the concept transmitted in the *Computus Digbaeanus* and the *Computus Cottonianus* have parallels with the general algorithm of this *argumentum*, I believe that the number of points transmitted in these two texts represent the original. They remained unaltered in β, and were only changed according to a different concept when copied in the *Fragmentum Nanciacense.* The deliberate omission of the fixed parameter 1 in β was retained in the *Fragmentum Nanciacense*, since this seems to have suited the computistical custom of the Irish copyist. In the *Computus Digbaeanus* and the *Computus Cottonianus*, however, this parameter was silently added, because their compilers, being probably trained in an Anglo-Saxon computistical milieu, followed the same practice as the author of the original *argumentum.* Since this silent addition was made in different parts of the calculation, it seems unlikely that *Argumentum XIII* of the *Computus Cottonianus* was directly copied from the *Computus Digbaeanus.* Hence, the content of these three versions suggests the following basic stemma (*Figure* 2), even though only a proper edition of *Argumentum XIII* can solve the question of their relation:

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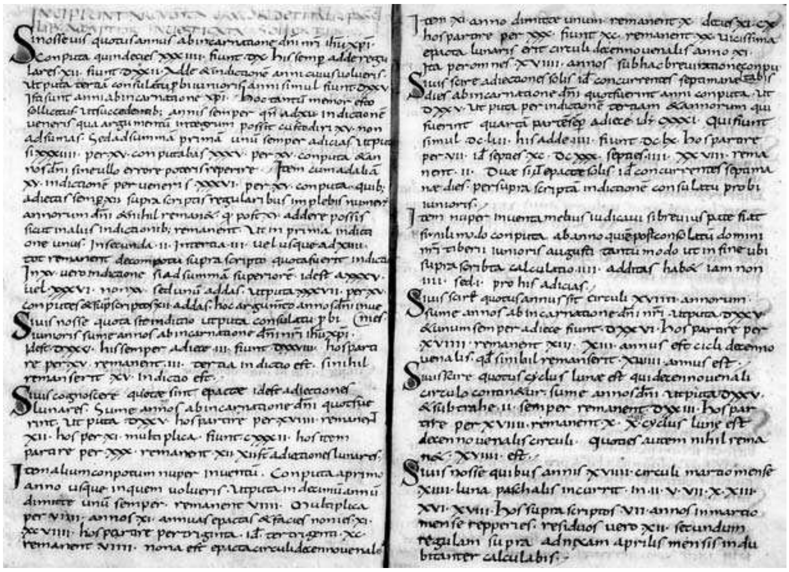
Figure 2 Tentative stemma of the early transmission of *Argumentum XIII.*

Appendix III: Facsimiles



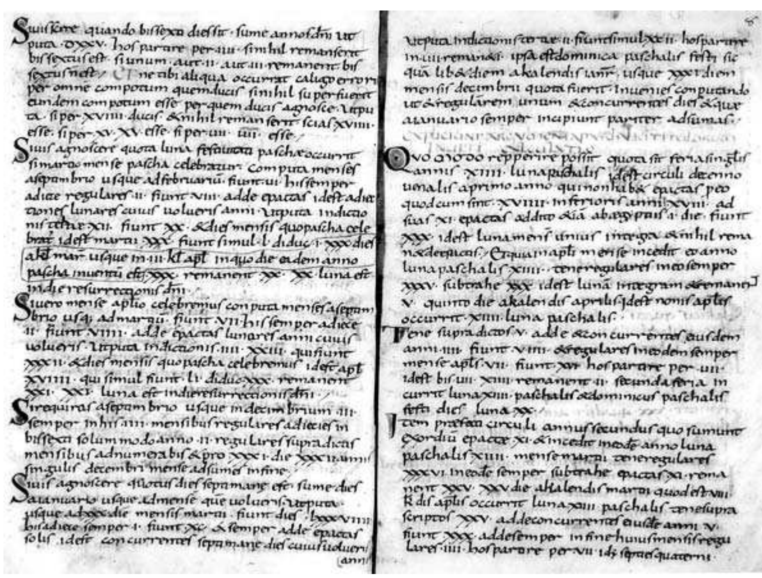


Plate 1 *Argumenta I–X* as a strictly defined corpus with *Argumentum XIV* attached as found in Group B texts (here MS Vatican, Biblioteca Apostolica, Pal. Lat. 1447, 6v–8r). *(Continued)*

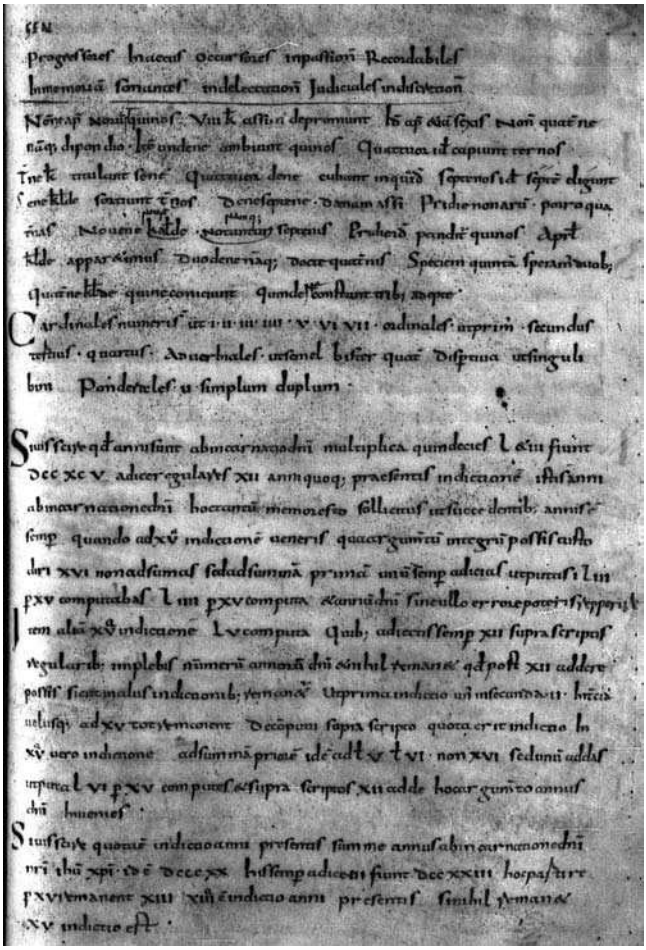


Plate 1 *(Continued)*

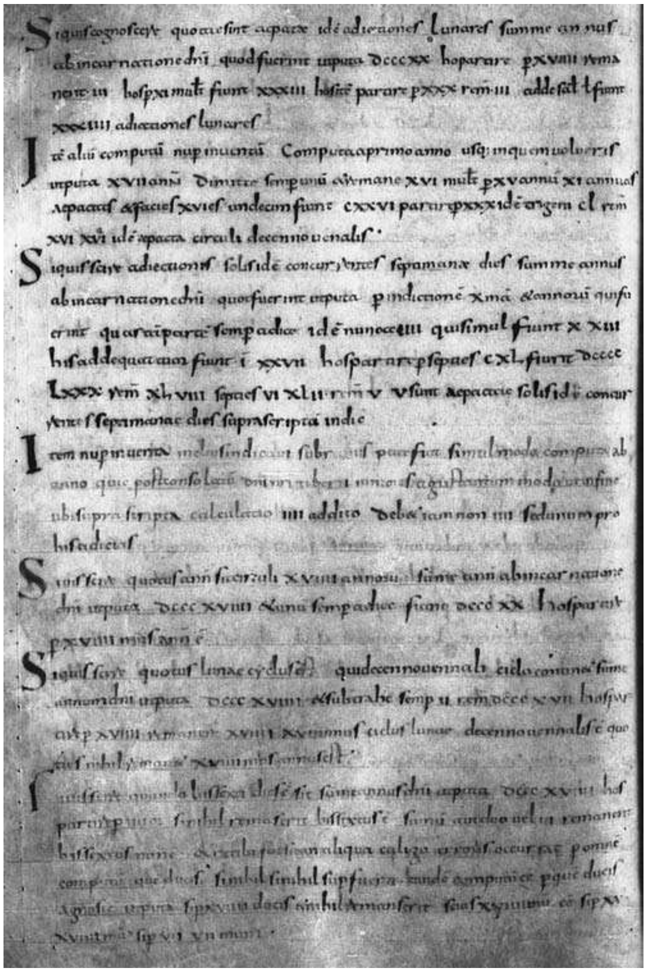


Plate 1 *(Continued)*

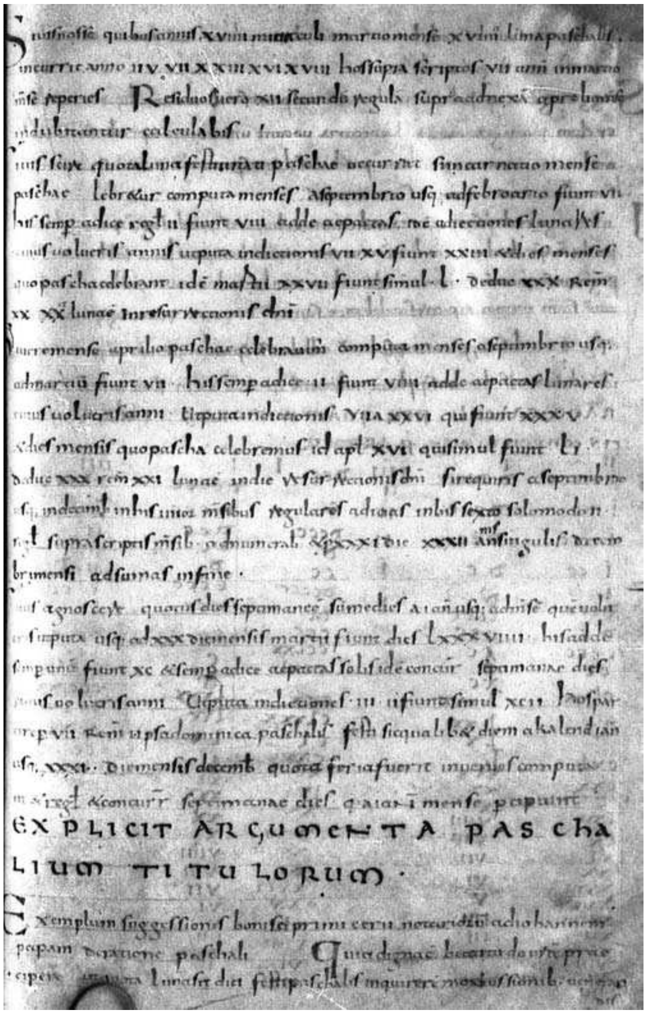


Plate 2 The *Computus Parisinus* of AD 819/20 (MS Paris, Bibliothèque Nationale, Nouvelle acquisition 1615, 154r–155r). *(Continued)*

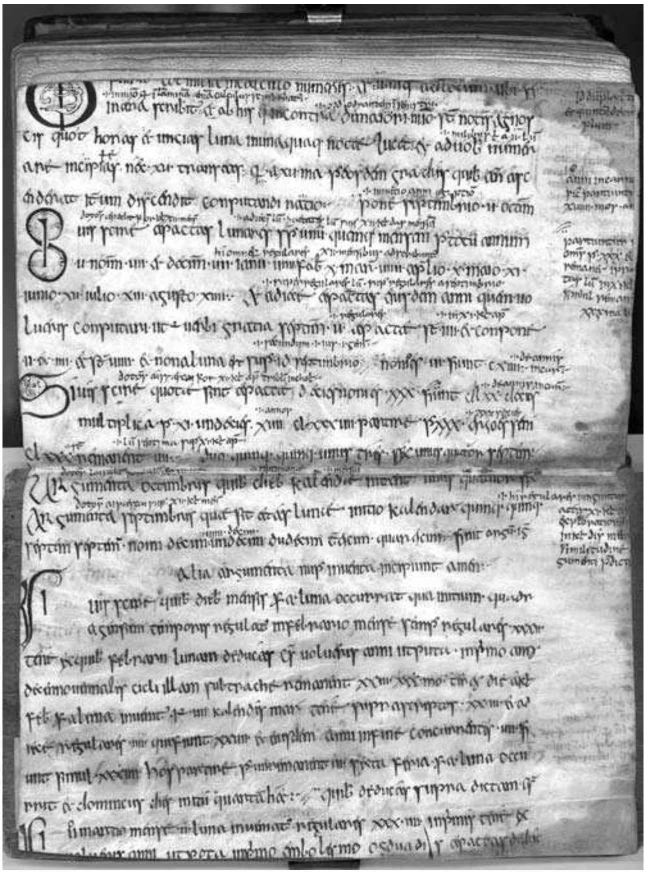


Plate 2 *(Continued)*

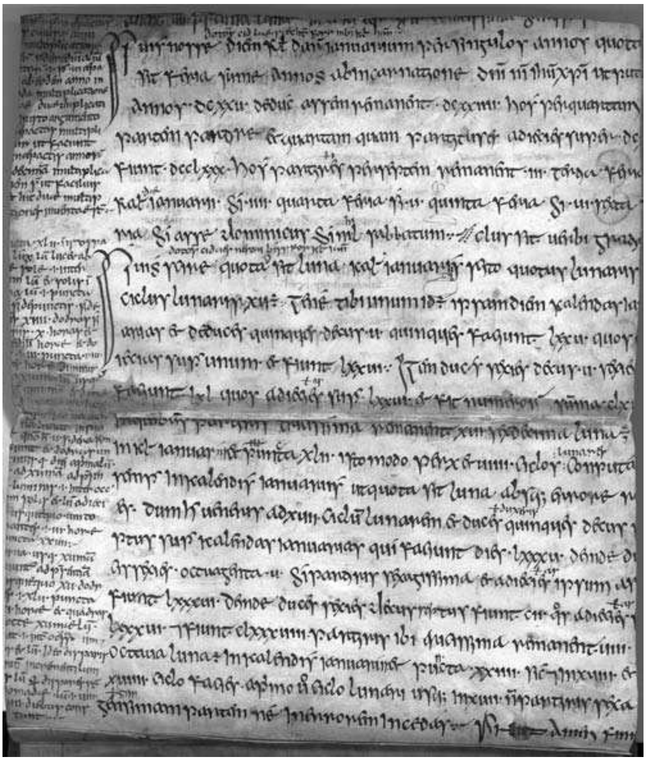


Plate 3 The *Fragmentum Nanciacense* (MS Nancy, Bibliothèque municipale, 317 (356)).